



GCE

Mathematics (MEI)

Unit 4751: Introduction to Advanced Mathematics (C1)

Advanced Subsidiary GCE

Mark Scheme for June 2014

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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1. These are the annotations, (including abbreviations), including those used in scoris, which are used when marking

Annotation in scoris	Meaning
BP	Blank Page – this annotation must be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓ and *	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance	
1	(i)	$\frac{1}{9}$	2 [2]	isw conversion to decimal M1 for 9 or for 3^{-2} or for $\frac{1}{3}$ Except M0 for 9 from $27/3$ or $\sqrt[3]{27}$	ie M1 for evidence of $(\sqrt[3]{27})^2$ or $1/(\sqrt[3]{27})$ found correctly
1	(ii)	$2a^2c^{-4}$ or $\frac{2a^2}{c^4}$ as final answer	3 [3]	B1 for each element; must be multiplied if B0, allow SC1 for $64a^6c^3$ obtained from numerator or for all elements correct but added	
2		midpt M of AB = $\left(\frac{1+6}{2}, \frac{5-1}{2}\right)$ oe isw soi subst of their midpt into $y = 2x - 5$ and attempting to evaluate all work correct and 'Yes' oe	M1 M1 A1 [3]	condone lack of brackets; accept in the form $x = 7/2$ oe, $y = 2$ oe eg $2 \times$ their $3.5 - 5 =$ their result accept $2 = 2 \times 3.5 - 5$	<u>alt methods:</u> allow 2 nd M1 for finding correct eqn of AB as $y = -\frac{6x}{5} + \frac{31}{5}$ oe and attempting to solve as simult eqn with $y = 2x - 5$ for x or y or allow M1 for finding in unsimplified form the eqn of the line through their midpt with gradient 2 and A1 for showing it is $y = 2x - 5$, so Yes
3	(i)	graph of shape with vertices at $(-2, -3)$, $(0, 0)$ and $(2, -4)$	2	M1 for 2 vertices correct	condone lines unruled; condone just missing vertex: $\frac{1}{4}$ grid square tolerance

Question		Answer	Marks	Guidance	
			[2]		
3	(ii)	graph of shape with vertices at $(1, -1)$, $(3, 2)$ and $(5, -2)$	2 [2]	M1 for 2 vertices correct or for shape with vertices at $(-5, -1)$, $(-3, 2)$ and $(-1, -2)$ condone lines unruled; condone just missing vertex: $\frac{1}{4}$ grid square tolerance	
4	(i)	$61 - 28\sqrt{3}$	3 [3]	B2 for 61 or B1 for $49 + 12$ found in expansion (may be in a grid) and B1 for $-28\sqrt{3}$ if B0, allow M1 for at least three terms correct in $49 - 14\sqrt{3} - 14\sqrt{3} + 12$ the correct answer obtained then spoilt earns SC2 only	
4	(ii)	$4\sqrt{3}$	2 [2]	M1 for $\sqrt{50} = 5\sqrt{2}$ or $\sqrt{300} = 10\sqrt{3}$ or $20\sqrt{300} = 200\sqrt{3}$ or $\sqrt{48} = 2\sqrt{12}$ seen	

Question		Answer	Marks	Guidance	
5		$3a + 12 [=ac + 5f]$ $3a - ac = 5f - 12$ or ft $a(3 - c) = 5f - 12$ or ft $[a =] \frac{5f - 12}{3 - c}$ oe or ft as final answer	M1 M1 M1 M1 [4]	for expanding brackets correctly for collecting a terms on one side, remaining terms on other for factorising a terms; may be implied by final answer for division by their two-term factor; for all 4 marks to be earned, work must be fully correct	annotate this question if partially correct ft only if two a terms ft only if two a terms, needing factorising may be earned before 2 nd M1
6		$(3x + 1)(x + 3)$ $x < -3$ [or] $x > -1/3$ oe	M1 A1 A1 [3]	or $3(x + 1/3)(x + 3)$ or for $-1/3$ and -3 found as endpoints eg by use of formula mark final answers; allow only A1 for $-3 > x > -1/3$ oe as final answer or for $x \leq -3$ and $x \geq -1/3$ if M0, allow SC1 for sketch of parabola the right way up with their solns ft their endpoints	A0 for combinations with only one part correct eg $-3 > x < -1/3$, though this would earn M1 if not already awarded

Question		Answer	Marks	Guidance
7		70 000 www	4 [4]	<p>throughout, condone xs included eg $(2x)^4$</p> <p>M3 for $35 \times 5^3 \times 2^4$ oe</p> <p>or M2 for two of these elements multiplied</p> <p>or M1 for 35 oe or for 1 7 21 35 35 21 7 1 row of Pascal's triangle seen</p> <p>annotate this question if partially correct allow 4 for $70 000x^4$ www; may also include other terms in expansion. Allow marks even if wrong term selected; mark the coefficient of x^4 may be unsimplified, but do not allow 35 in factorial form unless evaluated later or for all three elements seen together (eg in table) but not multiplied</p>

Question		Answer	Marks	Guidance	
8		use of $f(2)$ $4 \times 2^3 + 2k + 6 = 42$ $k = 2$ $[x =] -1$	M1 M1 A1 A1 [4]	2 substituted in $f(x)$ or $f(2) = 42$ seen or correct division of $4x^3 + kx + 6$ by $x - 2$ as far as obtaining $4x^2 + 8x + (k + 16)$ oe [may have $4x^2 + 8x + 18$] or $6 + 2(k + 16) = 42$ oe or finding (usually after division) that the constant term is 36 and then working with the x term to find k eg $kx + 16x = 18x$ as their answer, not just a trial; A0 for just $f(-1) = 0$ with no further statement A0 if confusion between roots and factors in final statement eg 'x + 1 is a root', even if they also state $x = -1$	accept with no working since it can be found by inspection
9	(i)	$3n^2 + 6n + 5$ isw	B2 [2]	M1 for a correct expansion of at least one of $(n + 1)^2$ and $(n + 2)^2$	

Question		Answer	Marks	Guidance	
9	(ii)	odd numbers with valid explanation	B2 [2]	<p>marks dep on 9(i) correct or starting again</p> <p>for B2 must see at least odd \times odd = odd [for $3n^2$] (or when n is odd, $[3]n^2$ is odd) and odd [+ even] + odd = even soi,</p> <p>condone lack of odd \times even = even for $6n$;</p> <p>condone no consideration of n being even</p> <p>or B2 for deductive argument such as: $6n$ is always even [and 5 is odd] so $3n^2$ must be odd so n is odd</p> <p>B1 for odd numbers with a correct partial explanation or a partially correct explanation</p> <p>or B1 for an otherwise fully correct argument for odd numbers but with conclusion positive odd numbers or conclusion negative odd numbers</p> <p>B0 for just a few trials and conclusion</p>	<p>accept a full valid argument using odd and even from starting again</p> <p>Ignore numerical trials or examples in this part – only a generalised argument can gain credit</p>

Question		Answer	Marks	Guidance	
10	(i)	(7, 0)	1 [1]	accept $x = 7, y = 0$	condone 7, 0
10	(ii)	$\sqrt{13}$ $(x - 4)^2 + (y - 2)^2 = 13$ or ft their evaluated r^2 , isw	2 2 [4]	M1 for Pythag used correctly eg $[r^2 =] 3^2 + 2^2$ or for subst A or their B in $(x - 4)^2 + (y - 2)^2 [= r^2]$ or B1 for $[r =] \pm\sqrt{13}$ M1 for one side correct, as part of an equation with x and y terms	annotate this question if partially correct allow recovery if some confusion between squares and roots but correct answer found do not accept $(\sqrt{13})^2$ instead of 13; allow M1 for LHS for $(x - 4)^2 + (y - 2)^2 = r^2$ (or worse, $(x - 4)^2 + (y - 2)^2 = r$) (may be seen in attempt to find radius)
10	(iii)	(7, 4)	2 [2]	B1 each coord accept $x = 7, y = 4$ if B0, then M1 for a vector or coordinates approach such as '3 along and 2 up' to get from A to C oe or M1 for $\frac{x_D + 1}{2} = 4$ and $\frac{y_D [+0]}{2} = 2$	condone 7, 4 or M1 for longer method, finding the equation of the line CD as $y = 2/3(x - 1)$ oe <u>and</u> then attempting to find intn with their circle

Question		Answer	Marks	Guidance	
10	(iv)	<p>grad tgt = $-3/2$ oe</p> <p>$y - \text{their } 4 = \text{their } (-3/2)(x - \text{their } 7)$</p> <p>$y = -1.5x + 14.5$ oe isw</p>	M2 M1 A1 [4]	<p>correctly obtained or ft their D if used</p> <p>M1 for grad AD = $\frac{4-0}{7-1}$ oe isw or $2/3$ oe seen or used in this part or for their grad tgt = -1/ their grad AD</p> <p>or subst (7, 4) into $y = \text{their } (-3/2)x + b$</p> <p>M0 if grad AD oe used or if a wrong gradient appears with no previous working</p> <p>must be in form $y = ax + b$</p>	<p>annotate this question if partially correct may use AD, CD or AC</p> <p>NB grad AD etc may have been found in (iii). allow marks if used in this part – mark the copy of (iii) that appears below the image for (iv)</p> <p>condone $y = \frac{-3x + 29}{2}$</p> <p>condone $y = -1.5x + b$ and $b = 14.5$ oe</p>

Question		Answer	Marks	Guidance	
11	(i)	$x = 4$ $(4, -3)$	B1 B1 [2]	or $x = 4, y = -3$	condone 4, -3
11	(ii)	$(0, 13)$ isw [when $y = 0$,] $(x - 4)^2 = 3$ $[x =] 4 \pm \sqrt{3}$ or $\frac{8 \pm \sqrt{12}}{2}$ isw	1 M1 A2 [4]	or [when $x = 0$], $y = 13$ isw 0 for just $(13, 0)$ or $(k, 13)$ where $k \neq 0$ or $x^2 - 8x + 13 [= 0]$ need not go on to give coordinate form A1 for one root correct	annotate this question if partially correct may be implied by correct value(s) for x found allow M1 for $y = x^2 - 8x + 13$ only if they go on to find values for x as if y were 0
11	(iii)	replacement of x in their eqn by $(x - 2)$ completion to given answer $y = x^2 - 12x + 33$, showing at least one correct interim step	M1 A1 [2]	may be simplified; eg $[y =] (x - 6)^2 - 3$ or allow M1 for $(x - 6 - \sqrt{3})(x - 6 + \sqrt{3}) [= 0 \text{ or } y]$ cao; condone using $f(x - 2)$ in place of y	condone omission of 'y =' for M1, but must be present in final line for A1

Question		Answer	Marks	Guidance
11	(iv)	$x^2 - 12x + 33 = 8 - 2x$ or $(x - 6)^2 - 3 = 8 - 2x$	M1	for equating curve and line; correct eqns only; or for attempt to subst $(8 - y)/2$ for x in $y = x^2 - 12x + 33$
		$x^2 - 10x + 25 = 0$	M1	for rearrangement to zero, condoning one error such as omission of ' $= 0$ '
		$(x - 5)^2 [= 0]$	A1	or showing $b^2 = 4ac$
		$x = 5$ www [so just one point of contact]	A1	may be part of coordinates $(5, k)$
		point of contact at $(5, -2)$	A1	dependent on previous A1 earned; allow for $y = -2$ found
		<u>alt. method</u>	or	examiners: use one mark scheme or the other, to the benefit of the candidate if both methods attempted, but do not use a mixture of the schemes
		for curve, $y' = 2x - 12$	M1	
		$2x - 12 = -2$	M1	for equating their y' to -2
		$x = 5$, and y shown to be -2 using eqn to curve	A1	
		tgt is $y + 2 = -2(x - 5)$	A1	
		deriving $y = 8 - 2x$	A1	condone no further interim step if all working in this part is correct so far
			[5]	

Question		Answer	Marks	Guidance	
12	(i)	$y = (x + 5)(x + 2)(2x - 3)$ or $y = 2(x + 5)(x + 2)(x - 3/2)$	2 [2]	M1 for $y = (x + 5)(x + 2)(x - 3/2)$ or $(x + 5)(x + 2)(2x - 3)$ with no equation or $(x + 5)(x + 2)(2x - 3) = 0$ but M0 for $y = (x + 5)(x + 2)(2x - 3) - 30$ or $(x + 5)(x + 2)(2x - 3) = 30$ etc	allow 'f(x) =' instead of 'y =' ignore further work towards (ii) but do not award marks for (i) in (ii)
12	(ii)	correct expansion of a pair of their linear two-term factors ft isw correct expansion of the correct linear and quadratic factors and completion to given answer $y = 2x^3 + 11x^2 - x - 30$	M1 M1 [2]	ft their factors from (i); need not be simplified; may be seen in a grid must be working for this step before given answer or for direct expansion of all three factors, allow M2 for $2x^3 + 10x^2 + 4x^2 - 3x^2 + 20x - 15x - 6x - 30$ oe (M1 if one error) or M1M0 for a correct direct expansion of $(x + 5)(x + 2)(x - 3/2)$ condone lack of brackets if used as if they were there	allow only first M1 for expansion if their (i) has an extra -30 etc do not award 2 nd mark if only had $(x - 3/2)$ in (i) and suddenly doubles RHS at this stage condone omission of 'y =' or inclusion of '= 0' for this second mark (some cards have already lost a mark for that in (i)) allow marks if this work has been done in (i) – mark the copy of (i) that appears below the image for (ii)

Question		Answer	Marks	Guidance	
12	(iii)	<p>ruled line drawn through $(-2, 0)$ and $(0, 10)$ and long enough to intersect curve at least twice</p> <p>-5.3 to -5.4 and 1.8 to 1.9</p>	<p>B1</p> <p>B2</p> <p>[3]</p>	<p>tolerance half a small square on grid at $(-2, 0)$ and $(0, 10)$</p> <p>B1 for one correct ignore the solution -2 but allow B1 for both values correct but one extra or for wrong 'coordinate' form such as $(1.8, -5.3)$</p>	<p>insert BP on spare copy of graph if not used, to indicate seen – this is included as part of image, so scroll down to see it</p> <p>accept in coordinate form ignoring any y coordinates given;</p>
12	(iv)	$2x^3 + 11x^2 - x - 30 = 5x + 10$ $2x^3 + 11x^2 - 6x - 40 [= 0]$ <p>division by $(x + 2)$ and correctly obtaining $2x^2 + 7x - 20$</p> <p>substitution into quadratic formula or for completing the square used as far as</p> $\left(x + \frac{7}{4}\right)^2 = \frac{209}{16} \text{ oe}$ $[x =] \frac{-7 \pm \sqrt{209}}{4} \text{ oe isw}$	<p>M1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>for equating curve and line; correct eqns only</p> <p>for rearrangement to zero, condoning one error</p> <p>or showing that $(x + 2)(2x^2 + 7x - 20) = 2x^3 + 11x^2 - 6x - 40$, with supporting working</p> <p>condone one error eg a used as 1 not 2, or one error in the formula, using given $2x^2 + 7x - 20 = 0$</p> <p>dependent only on 4th M1</p>	annotate this question if partially correct

Appendix

For candidates who have done work in the wrong section of the answer book (usually continuing the next part in the same section instead of moving on): please consult your TL, except in the case of 12ii done in 12i, which has already been dealt with in the scheme.

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