



**GCE**

**Mathematics (MEI)**

Unit **4754A**: Applications of Advanced Mathematics: Paper A

Advanced GCE

**Mark Scheme for June 2014**

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

© OCR 2014

1. These are the annotations, (including abbreviations), including those used in scoris, which are used when marking

Annotation in scoris	Meaning
<b>BP</b>	Blank Page – this annotation <b>must</b> be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓ and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

## 2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

- a Annotations should be used whenever appropriate during your marking.

**The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks.** It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

### **M**

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

### **A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

### **B**

Mark for a correct result or statement independent of Method marks.

**E**

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep \*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

- h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question	Answer	Marks	Guidance
1	$\frac{3x}{(2-x)(4+x^2)} = \frac{A}{2-x} + \frac{Bx+C}{4+x^2}$ $\Rightarrow 3x = A(4+x^2) + (Bx+C)(2-x)$ $x=2 \Rightarrow 6 = 8A, A = \frac{3}{4}$ $x^2 \text{ coeffs: } 0 = A - B \Rightarrow B = \frac{3}{4}$ $\text{constants: } 0 = 4A + 2C \Rightarrow C = -1\frac{1}{2}$	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[5]</p>	<p>correct form of partial fractions (condone additional coeffs eg <math>\frac{Ax+B}{2-x} + \frac{Cx+D}{4+x^2}</math> * for M1 <b>BUT</b> <math>\frac{A}{2-x} + \frac{B}{4+x^2}</math> ** is M0 )</p> <p>Multiplying through oe and substituting values or equating coeffs at <b>LEAST AS FAR AS FINDING A VALUE</b> for one of their unknowns (even if incorrect) Can award in cases * and ** above Condone a sign error or single computational error for M1 but not a conceptual error Eg <math>3x=A(2-x)+(Bx+C)(4+x^2)</math> is M0 <math>3x(2-x)(4+x^2)=A(4+x^2)+(Bx+C)(2-x)</math> is M0 Do not condone missing brackets unless it is clear from subsequent work that they were implied. Eg <math>3x=A(4+x^2) + Bx+C(2-x)=4A+Ax^2+Bx+2C-Cx</math> is M0 <math>= 4A+Ax^2 + 2Bx-Bx^2+2C-Cx</math> is M1</p> <p>oe www [SC B1 <math>A=3/4</math> from cover up rule can be applied, then the M1 applies to the other coefficients] <b>NB</b> <math>\frac{A}{2-x} + \frac{B}{4+x^2} \Rightarrow A=3/4</math> is A0 ww (wrong working)</p> <p>oe www</p> <p>oe www [In the case of * above, all 4 constants are needed for the final A1] Ignore subsequent errors when recompiling the final solution provided that the coeffs were all correct</p>
2	$(4+x)^{\frac{3}{2}} = 4^{\frac{3}{2}} \left(1 + \frac{1}{4}x\right)^{\frac{3}{2}}$	M1	dealing with the '4' to obtain $4^{3/2} \left(1 + \frac{x}{4}\right)^{3/2}$

Question	Answer	Marks	Guidance
	$= 8\left(1 + \frac{3}{2}\left(\frac{1}{4}x\right) + \frac{3}{2} \cdot \frac{1}{2} \cdot \frac{1}{2!}\left(\frac{1}{4}x\right)^2 + \dots\right)$ $= 8 + 3x$ $+ 3/16 x^2$ <p>Valid for <math>-4 &lt; x &lt; 4</math> or <math> x  &lt; 4</math></p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>[5]</p>	<p>(or expanding as <math>4^{3/2} + \frac{3}{2}4^{1/2}x + \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)4^{-1/2}\frac{x^2}{2!} + \dots</math> and having all the powers of 4 correct)</p> <p>correct <b>binomial coeffs</b> for <math>n=3/2</math> ie 1, 3/2, 3/2.1/2.1/2! Not nCr form</p> <p>Indep of coeff of x</p> <p>Indep of first M1</p> <p>8+3x                      www</p> <p>...+3/16 x<sup>2</sup>                      www</p> <p>Ignore subsequent terms</p> <p>accept <math>\leq</math> s or a combination of <math>&lt;</math> and <math>\leq</math> , but not <math>-4 &gt; x &gt; 4</math>, <math> x  &gt; 4</math>, or say <math>-4 &lt; x</math></p> <p>condone <math>-4 &lt;  x  &lt; 4</math></p> <p>Indep of all other marks</p> <p>Allow MR throughout this question for <math>n=m/2</math> where <math>m \in \mathbb{N}</math>, and m odd and then -1 MR provided it is at least as difficult as the original.</p>



Question			Answer	Marks	Guidance											
3	(i)		<table><tr><td>x</td><td>0</td><td>0.1963</td><td>0.3927</td><td>0.5890</td><td>0.7854</td></tr><tr><td>y</td><td>0</td><td>0.4493</td><td>0.6792</td><td>0.9498</td><td>1.3254</td></tr></table>  $A = (\pi/32) [(0 + 1.3254) + 2( 0.4493 + 0.6792 + 0.9498)]$  $= 0.538$	x	0	0.1963	0.3927	0.5890	0.7854	y	0	0.4493	0.6792	0.9498	1.3254	B2,1,0   M1  A1  <
x	0	0.1963	0.3927	0.5890	0.7854											
y	0	0.4493	0.6792	0.9498	1.3254											

Question	Answer	Marks	Guidance
4 (i)	<p>EITHER Use of <math>\cos=1/\sec</math> (or <math>\sin=1/\operatorname{cosec}</math>)</p> <p>From RHS</p> $\frac{1 - \tan \alpha \tan \beta}{\sec \alpha \sec \beta}$ $= \frac{1 - \sin \alpha / \cos \alpha \cdot \sin \beta / \cos \beta}{1 / \cos \alpha \cdot 1 / \cos \beta}$ $= \cos \alpha \cos \beta \left(1 - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}\right)$ $= \cos \alpha \cos \beta - \sin \alpha \sin \beta$ $= \cos(\alpha + \beta)$	<p>B1</p> <p>M1</p> <p>A1</p>	<p>Must be <b>used</b></p> <p>Substituting and simplifying as far as having no fractions within a fraction</p> <p>[need more than <math>\frac{1-tt}{\sec \sec} = cc - ss</math> ie an intermediate step that can lead to cc-ss]</p> <p>Convincing simplification and correct use of <math>\cos(\alpha+\beta)</math></p> <p><b>Answer given</b></p>
	<p>OR From LHS, <math>\cos=1/\sec</math> or <math>\sin=1/\operatorname{cosec}</math> <b>used</b></p> $\cos(\alpha + \beta)$ $= \cos \alpha \cos \beta - \sin \alpha \sin \beta$ $= \frac{1}{\sec \alpha \sec \beta} - \sin \alpha \sin \beta$ $= \frac{1 - \sec \alpha \sin \alpha \sec \beta \sin \beta}{\sec \alpha \sec \beta}$ $= \frac{1 - \tan \alpha \tan \beta}{\sec \alpha \sec \beta}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>Correct angle formula and substitution and simplification to one term</p> <p>OR eg <math>\cos \alpha \cos \beta - \sin \alpha \sin \beta</math>  <math>= \cos \alpha \cos \beta (1 - \tan \alpha \tan \beta)</math></p> <p>Simplifying to final answer www</p> <p><b>Answer given</b></p> <p>Or any equivalent work but must have more than cc-ss=answer.</p>

Question			Answer	Marks	Guidance
4	(ii)		$\beta = \alpha$ $\cos 2\alpha = \frac{1 - \tan^2 \alpha}{\sec^2 \alpha}$ $= \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha}$ ..... OR, without Hence, $\cos 2\alpha = \cos^2 \alpha \left(1 - \frac{\sin^2 \alpha}{\cos^2 \alpha}\right) = \frac{1}{\sec^2 \alpha} (1 - \tan^2 \alpha) = \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha}$	M1  A1  ..... M1 A1  <b>[2]</b>	$\beta = \alpha$ used , Need to see $\sec^2 \alpha$  Use of $\sec^2 \alpha = 1 + \tan^2 \alpha$ to give required result <b>Answer Given</b> ..... Use of $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$ soi Simplifying and using $\sec^2 \alpha = 1 + \tan^2 \alpha$ to final answer <b>Answer Given</b> Accept working in reverse to show RHS=LHS, or showing equivalent
			$\cos 2\theta = \frac{1}{2}$  i. $2\theta = 60^\circ, 300^\circ, \theta = 30^\circ, 150^\circ$	M1  A1 A1  <b>[3]</b>	Soi or from $\tan^2 \theta = 1/3$ oe from $\sin^2 \theta$ or $\cos^2 \theta$  First correct solution Second correct solution and no others in the range SC B1 for $\pi/6$ and $5\pi/6$ and no others in the range

Question		Answer	Marks	Guidance
5	(i)	<p>EITHER</p> $x = e^{3t}, y = te^{2t}$ $dy/dt = 2te^{2t} + e^{2t}$ $\Rightarrow dy/dx = (2te^{2t} + e^{2t})/3e^{3t}$ <p>when <math>t = 1</math>, <math>dy/dx = 3e^2/3e^3 = 1/e</math></p> <p>.....</p> <p>OR</p> $3t = \ln x, y = \frac{\ln x}{3} e^{2/3 \ln x} = \frac{x^{2/3} \ln x}{3}$ $dy/dx = \frac{1}{3} x^{2/3} \frac{1}{x} + \ln x \frac{2}{9} x^{-1/3}$ $= \frac{1}{3e^t} + \frac{2t}{3e^t}$ $dy/dx = 1/3e + 2/3e = 1/e$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>.</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p><b>[4]</b></p>	<p>soi</p> <p>Their <math>dy/dt \div dx/dt</math> in terms of <math>t</math></p> <p>oe cao allow for unsimplified form even if subsequently cancelled incorrectly ie can isw</p> <p>cao www must be simplified to <math>1/e</math> oe</p> <p>.....</p> <p>Any equivalent form of <math>y</math> in terms of <math>x</math> only</p> <p>Differentiating their <math>y</math> provided not eased ie need a product including</p> <p>In <math>kx</math> and <math>x^p</math> and subst <math>x = e^{3t}</math> to obtain <math>dy/dx</math> in terms of <math>t</math></p> <p>oe cao</p> <p>www cao <b>exact only</b> must be simplified to <math>1/e</math> or <math>e^{-1}</math></p>
		<p><math>3t = \ln x \Rightarrow t = (\ln x)/3</math></p> <p>ii. <math>y = (\ln x) / 3e^{(2 \ln x)/3}</math></p> <p>iii. <math>= \frac{1}{3} x^{\frac{2}{3}} \ln x</math></p>	<p>B1</p> <p>M1</p> <p>A1</p> <p><b>[3]</b></p>	<p>Finding <math>t</math> correctly in terms of <math>x</math></p> <p>Subst in <math>y</math> using their <math>t</math></p> <p>Required form <math>ax^b \ln x</math> only</p> <p>NB If this work was already done in 5(i), marks can only be scored in 5(ii) if candidate specifically refers in this part to their part (i).</p>

Question	Answer	Marks	Guidance
6	$y = (1 + 2x^2)^{\frac{1}{3}} \Rightarrow y^3 = 1 + 2x^2$ $\Rightarrow x^2 = \frac{1}{2}(y^3 - 1)$ $V = \int_1^2 \pi x^2 \, dy = \frac{1}{2} \pi \int_1^2 (y^3 - 1) \, dy$ $= \frac{1}{2} \pi \left[ \frac{1}{4} y^4 - y \right]_1^2 = \frac{1}{2} \pi \left( 2 + \frac{3}{4} \right)$ $= \frac{11}{8} \pi$	<p>M1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p><b>[6]</b></p>	<p>finding <math>x^2</math> (or <math>x</math>) correctly in terms of <math>y</math></p> <p>For M1 need <math>\int \pi x^2 \, dy</math> with substitution for their <math>x^2</math> (in terms of <math>y</math> only),</p> <p>Condone absence of <math>dy</math> throughout if intention clear. (need <math>\pi</math>) www For A1 it must be correct with correct limits 1 and 2, but they may appear later</p> <p><math>1/2[y^4/4 - y]</math> independent of <math>\pi</math> and limits</p> <p>substituting both their limits in correct order in correct expression, condone a minor slip for M1 (if using <math>y=0</math> as lower limit then '-0' is enough)</p> <p>condone absence of <math>\pi</math> for M1</p> <p>oe exact only www (<math>1\frac{3}{8} \pi</math> or <math>1.375\pi</math>)</p>

Question	Answer	Marks	Guidance
7 (i)	$AB = \sqrt{5^2 + (-2)^2} = \sqrt{29}$ $AC = \sqrt{3^2 + 4^2} = 5$ $\cos \theta = \frac{\begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}}{\sqrt{29.5}} = \frac{15+0+0}{5\sqrt{29}} = 0.5571$ $\Rightarrow \theta = 56.15^\circ$ $\text{Area} = \frac{1}{2} \times 5 \times \sqrt{29} \times \sin \theta$ $= 11.18$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[7]</p>	<p>5.39 or better (condone sign error in vector for B1)</p> <p>Accept <math>\sqrt{25}</math> (condone sign error in vector for B1)</p> <p><math>\cos \theta = \frac{\text{scalar product of AB with AC}}{ AB  \cdot  AC }</math> (accept BA/CA)</p> <p><b>with substitution</b></p> <p>condone a single numerical error provided method is clearly understood</p> <p><b>[OR Cosine Rule, as far as <math>\cos \theta =</math> correct numerical expression]</b></p> <p><b>www</b> <math>\pm 0.5571, 0.557, 15/5\sqrt{29}, 15/\sqrt{25}\sqrt{29}</math> oe or better soi ( <math>\pm</math> for method only)</p> <p>www Accept answers that round to <math>56.1^\circ</math> or <math>56.2^\circ</math> or <math>0.98</math> radians (or better)</p> <p><b>NB vector <math>5i+0j+2k</math> leads to apparently correct answer but loses all A marks in part(i)</b></p> <p>Using their AB, AC, <math>\angle</math> CAB. Accept any valid method using trigonometry</p> <p>Accept <math>5\sqrt{5}</math> and answers that round to <math>11.18</math> or <math>11.19</math> (2dp) <b>www</b> or SCA1 for accurate work soi rounded at the last stage to <math>11.2</math> (but not from an incorrect answer, say from an incorrect angle or from say <math>11.17</math> or <math>11.22</math> stated and rounded to <math>11.2</math>) We will not accept inaccurate work from over rounded answers for the final mark.</p>

Question			Answer	Marks	Guidance
7	(ii)	(A)	$\overrightarrow{AB} \cdot \begin{pmatrix} 4 \\ -3 \\ 10 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -3 \\ 10 \end{pmatrix} = 5 \cdot 4 + 0 \cdot (-3) + (-2) \cdot 10 = 0$ $\overrightarrow{AC} \cdot \begin{pmatrix} 4 \\ -3 \\ 10 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -3 \\ 10 \end{pmatrix} = 3 \times 4 + 4 \times (-3) + 0 \times 10 = 0$	<p>B1</p> <p>B1</p> <p>[2]</p>	<p>Scalar product with one vector in the plane with numerical expansion shown.</p> <p>Scalar product, as above, with evaluation, with a second vector. <b>NB vectors are not unique</b></p> <p>SCB2 finding the equation of plane first by any valid method (or using vector product) and then clearly stating that the normal is proportional to the coefficients.</p> <p>SC For candidates who substitute all three points in the plane <math>4x-3y+10z=c</math> and show that they give the same result, award M1 If they include a statement explaining why this means that <math>4\mathbf{i}-3\mathbf{j}+10\mathbf{k}</math> is normal they can gain A1.</p>
7	(ii)	(B)	$4x-3y+10z=c$ $\Rightarrow 4x-3y+10z+12=0$	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>Required form <b>and</b> substituting the co-ordinates of a point on the plane</p> <p>oe If found in (A) it must be clearly referred to in (B) to gain the marks. Do not accept vector equation of the plane, as 'Hence'.</p> <p><math>4\mathbf{i}-3\mathbf{j}+10\mathbf{k} = -12</math> is M1A0</p>

Question			Answer	Marks	Guidance
7	(iii)		$\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix}$ $+ \lambda \begin{pmatrix} 4 \\ -3 \\ 10 \end{pmatrix}$ <p>Meets <math>4x - 3y + 10z + 12 = 0</math> when</p> $16\lambda - 3(4 - 3\lambda) + 10(5 + 10\lambda) + 12 = 0$ $\Rightarrow 125\lambda = -50, \lambda = -0.4$ <p>So meets plane ABC at <math>(-1.6, 5.2, 1)</math></p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p><b>[5]</b></p>	<p>Need <math>\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}</math></p> <p>oe</p> <p>Subst their <math>4\lambda, 4-3\lambda, 5+10\lambda</math> in equation of their plane from (ii)</p> <p><math>\lambda = -0.4</math> (NB not unique)</p> <p>cao www (condone vector)</p>
7	(iv)		<p>height = <math>\sqrt{(1.6^2 + (-1.2)^2 + 4^2)} = \sqrt{20}</math></p> <p>volume = <math>11.18 \times \sqrt{20} / 3 = 16.7</math></p>	<p>B1ft</p> <p>B1cao</p> <p><b>[2]</b></p>	<p>ft their (iii)</p> <p>50/3 or answers that round to 16.7 www and not from incorrect answers from (iii) ie not from say (1.6, 2.8, 9)</p>



Question		Answer	Marks	Guidance
8	(i)	<p>Either <math>h = (1 - \frac{1}{2} At)^2 \Rightarrow dh/dt = -A(1 - \frac{1}{2} At)</math>  <math>= -A\sqrt{h}</math>  when <math>t = 0</math>, <math>h = (1 - 0)^2 = 1</math> as required  .....</p> <p>Or <math>\int \frac{dh}{\sqrt{h}} = \int -A dt</math>  <math>2h^{1/2} = -At + c</math>  <math>h = \left(\frac{-At + c}{2}\right)^2</math> at <math>t=0</math>, <math>h=1</math>, <math>1=(c/2)^2 \Rightarrow c=2</math>, <math>h=(1-At/2)^2</math></p>	<p>M1 A1 B1 .....</p> <p>M1 A1 B1 [3]</p>	<p>Including function of a function, need to see middle step AG .....</p> <p>Separating variables correctly and integrating Including c. [Condone change of c.] Using initial conditions AG</p>
		<p>When <math>t = 20</math>, <math>h = 0</math>  <math>\Rightarrow 1 - 10A = 0</math>, <math>A = 0.1</math>  When the depth is 0.5 m, <math>0.5 = (1 - 0.05t)^2</math>  <math>\Rightarrow 1 - 0.05t = \sqrt{0.5}</math>, <math>t = (1 - \sqrt{0.5})/0.05 = 5.86\text{s}</math></p>	<p>M1 A1 M1 A1 [4]</p>	<p>Subst and solve for A cao substitute <math>h=0.5</math> and their A and solve for t www cao accept 5.9</p>

Question	Answer	Marks	Guidance
8 (iii)	$\frac{dh}{dt} = -B \frac{\sqrt{h}}{(1+h)^2}$ $\Rightarrow \int \frac{(1+h)^2}{\sqrt{h}} dh = - \int B dt$ <p>.....</p> <p><b>Either, LHS</b></p> $\int \frac{1+2h+h^2}{\sqrt{h}} dh$ $= \int (h^{-1/2} + 2h^{1/2} + h^{3/2}) dh$ <p>.....</p> <p><b>Or ,LHS, either,</b></p> $(1+2h+h^2)2h^{1/2} - \int 2h^{1/2}(2+2h)dh$ <p><b>or,</b> <math display="block">h^{1/2} + h^{3/2} + \frac{h^{5/2}}{3} + \int \frac{1}{2}h^{-3/2}(h+h^2+\frac{h^3}{3})dh</math></p> <p>.....</p> $2h^{1/2} + \frac{4h^{3/2}}{3} + \frac{2h^{5/2}}{5}$ $= -Bt + c$	<p>M1</p> <p>.....</p> <p>M1</p> <p>A1</p> <p>.....</p> <p>M1</p> <p>A1</p> <p>.....</p> <p>A1</p> <p>A1</p>	<p>separating variables correctly and intend to integrate <b>both sides</b> (may appear later) <b>[NB reading (1+h)<sup>2</sup> as 1+h<sup>2</sup> eases the question. Do not mark as a MR]</b> In cases where (1+h)<sup>2</sup> is MR as 1+h<sup>2</sup> or incorrectly expanded, as say 1+h+h<sup>2</sup> or 1+h<sup>2</sup>, allow first M1 for correct separation and attempt to integrate and can then score a max of M1M0A0A0A1 (for -Bt+c) A0A0, max 2/7.</p> <p>.....</p> <p>expanding (1+ h)<sup>2</sup> and dividing by <math>\sqrt{h}</math> to form a one line function of h (indep of first M1) with each term expressed as a single power of h eg must simplify say <math>1/\sqrt{h}+2h/\sqrt{h}+h^2\sqrt{h}</math>, condone a single error for M1</p> <p>(do not need to see integral signs)</p> <p><math>h^{-1/2} + 2h^{1/2} + h^{3/2}</math></p> <p>cao dep on second M only -do not need integral signs</p> <p>.....</p> <p>using <math>\int u dv = uv - \int v du</math> correct formula used correctly, indep of first M1 condone a single error for M1 if intention clear</p> <p>cao oe</p> <p>.....</p> <p>cao <b>oe, both sides dependent on first M1 mark</b></p> <p>cao need -Bt and c for second A1 but the constant may be on either side</p>

Question			Answer	Marks	Guidance
			$\Rightarrow 2h^{1/2} + 4h^{3/2}/3 + 2h^{5/2}/5 = -Bt + c$ When $t = 0, h = 1 \Rightarrow c = 56/15$  $\Rightarrow h^{1/2}(30 + 20h + 6h^2) = 56 - 15Bt$ *	A1  A1  <b>[7]</b>	from correct work only (accept 3.73 or rounded answers here but not for final A1) or $c = -56/15$ if constant on opposite side. <b>NB AG must be from all correct exact work including exact c.</b>
<b>8</b>	<b>(iv)</b>		$h = 0$ when $t = 20$ $\Rightarrow B = 56/300 = 0.187$ When $h = 0.5$ $56 - 2.8t = 29.3449...$ $\Rightarrow t = 9.52s$	M1 A1 M1 A1 <b>[4]</b>	Substituting $h=0, t=20$ Accept 0.187 Subst their $h=0.5$ , ft their $B$ and attempt to solve Accept answers that round to 9.5s www.

**OCR (Oxford Cambridge and RSA Examinations)**  
**1 Hills Road**  
**Cambridge**  
**CB1 2EU**

**OCR Customer Contact Centre**

**Education and Learning**

Telephone: 01223 553998

Facsimile: 01223 552627

Email: [general.qualifications@ocr.org.uk](mailto:general.qualifications@ocr.org.uk)

[www.ocr.org.uk](http://www.ocr.org.uk)

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

**Oxford Cambridge and RSA Examinations**  
**is a Company Limited by Guarantee**  
**Registered in England**  
**Registered Office; 1 Hills Road, Cambridge, CB1 2EU**  
**Registered Company Number: 3484466**  
**OCR is an exempt Charity**

**OCR (Oxford Cambridge and RSA Examinations)**  
**Head office**  
**Telephone: 01223 552552**  
**Facsimile: 01223 552553**

© OCR 2014

