OCR
Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4777**: Numerical Computation

Advanced GCE

Mark Scheme for June 2014

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

These are the annotations, (including abbreviations), including those used in scoris, which are used when marking

Annotation in scoris	**Meaning**
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

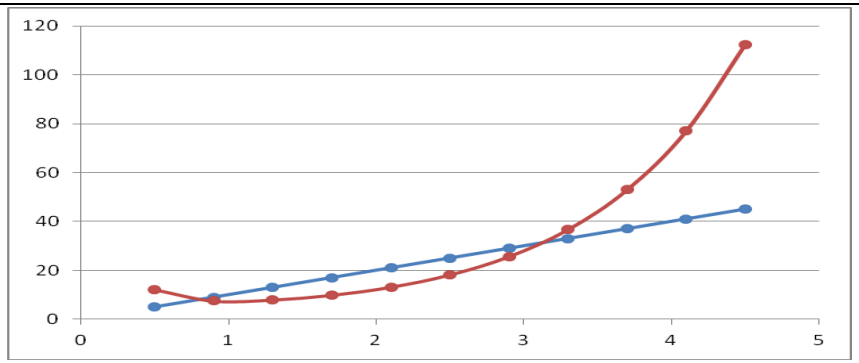
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		$-1 < g'(\alpha) < 1$	B1		
			Correct algebra to obtain given expression	M1		
			Derivative of RHS is $(1 - \lambda) + \lambda\, g'(x)$	B1		
			Set this to zero at $x = \alpha$ to obtain $\lambda = 1 / (1 - g'(\alpha))$	B1		
			In practice use $\lambda = 1 / (1 - g'(x_0))$	E1	Accept λ being re-set at each it'n	
				[5]		
1	(ii)			G2	Curve	
				G1	Line	
			For $x < 0$, LHS < 0 and RHS > 0 so no negative roots	E1		
			Evidence of convergence to smaller root , divergence from larger. Eg:		cao	
			1 3 3.2	M1A1	Setting up the iteration and using it from one starting point	
			0.738906 2.803162 3.353199			
			0.81032 2.35683 3.852941	A1A1	For using it for two more starting points	
			0.772453 1.613722 6.109826			
			0.790132 0.933161 53.03287			
			0.781264 0.742452 1.1E+22			
			0.785571 0.807959 #NUM!			
			0.783445 0.773415 #NUM!			
			0.784486 0.78962 #NUM!			
			0.783974 0.781505 #NUM!			
			0.784226 0.78545 #NUM!			
			Converging (to 0.784143)			

Question			Answer	Marks	Guidance	
			Set up and use the relaxed iteration. Eg: k: 3 10 3.131955 g'(x_0): 3.117202 2.491699 3.120372 lambda: 3.119725 –0.67037609 3.119859 3.119831 3.119837 3.119836 3.119836 3.119836	M1A1 M1A1 M1A1	Use g'(x_0) to calculate λ Set up relaxed iteration Use it to obtain correct root	
				[14]		
1	**(iii)**		k = 20 needs relaxation for both roots: k: 0.5 k: 4 20 0.53859578 20 4.216399 g'(x_0) 0.54212038 g'(x_0) 4.181296 –1.82737 0.54274725 3.286191 4.1920499 lambda: 0.54286338 lambda: 4.1889768 0.353685 0.54288504 –0.43741 4.1898748 0.54288908 4.189614 0.54288984 4.1896899 0.54288998 4.1896678 0.54289001 4.1896742 0.54289001 4.1896724 0.5429 (4 dp) 4.1897	M1A1 A1 A1 A1	Set up, g', lambda Smaller root Set up, g', lambda Larger root	
				[5]		

2 (i)

$Q = \Sigma (y - bx^2 - cx^3)^2$ — M1

Partial derivative of Q wrt b — M1A1

Set to zero and re-arrange convincingly to given expression — M1

Partial derivative of Q wrt c and set to zero — M1

Obtain $\Sigma x^2y = b\,\Sigma x^3 + c\,\Sigma x^4$ — A1

[6]

2 (ii)

x	y	xy	x^2y	x^2	x^3	x^4
1	7.24	7.24	7.24	1	1	1
2	12.15	24.3	48.6	4	8	16
3	13.84	41.52	124.56	9	27	81
4	12.25	49	196	16	64	256
5	7.07	35.35	176.75	25	125	625
		157.41	553.15	55	225	979

M1A1A1 Form sums

$55\ b + 225\ c = 157.41$

$225\ b + 979\ c = 553.15$

M1 Formulate equations

$-14.3111\ c = 22.19556$

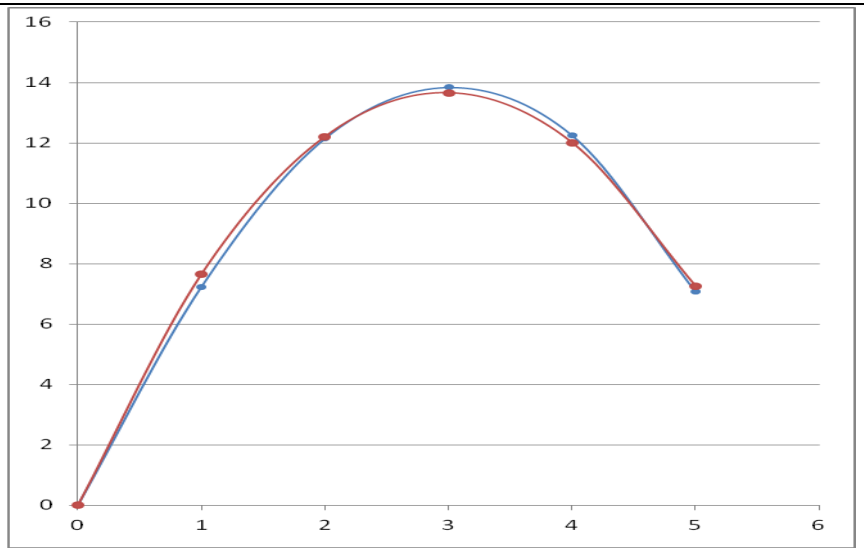
$c = -1.55093$

$b = 9.20672$

x	y	y-fitted	residual	resid^2
1	7.24	7.655789	0.415789	0.17288
2	12.15	12.20971	0.059714	0.003566
3	13.84	13.66178	–0.17822	0.031764
4	12.25	12.01198	–0.23802	0.056656
5	7.07	7.260311	0.190311	0.036218
			0.249565	**0.301084**

M1A1A1 Solve

M1A1A1 Find residuals, sum, sum of squares

Question			Answer	Marks	Guidance	
2	(ii) cont			G2 G2 **[14]**	Original data (higher at 3) Fitted curve (lower at 3)	
2	(iii)		Sum of residuals is not zero because there is no constant term. There is no constant term in order to give a curve through origin. The sum of squares of residuals would reduce because the fit would be better with one additional parameter.	E1 E1 E1 E1 **[4]**		
3	(i)		$T_n - I = A_2h^2 + A_4h^4 + A_6h^6 + \ldots$ $T_{2n} - I = A_2(h/2)^2 + A_4(h/2)^4 + A_6(h/2)^6 + \ldots$ $4(T_{2n} - I) - (T_n - I) = b_4h^4 + b_6h^6 + \ldots$ $4T_{2n} - T_n - 3I = b_4h^4 + b_6h^6 + \ldots$ $(4T_{2n} - T_n)/3 - I = B_4h^4 + B_6h^6 + \ldots$ $(T_n^* = (4T_{2n} - T_n)/3$ has error of order h^4 as given) $T_n^{**} = (16T_{2n}^* - T_n^*)/15$ has error of order h^6	M1 M1A1 M1A1 B1 **[6]**		

Question			Answer	Marks	Guidance	
3	(ii)		k: h x f(x) T T^* T^{**} 1 1.570796 0.000000 1.000000 1.570796 1.414214 1.896119 0.785398 0.785398 1.306563 1.974232 2.000269 0.392699 0.392699 1.175876 1.178097 1.387040 1.993570 2.000017 2.000000 0.196350 0.196350 1.093202 0.589049 1.247225 0.981748 1.353318 1.374447 1.407404 1.998393 2.000001 2.000000 0.098175 0.098175 1.047863 0.294524 1.135907 0.490874 1.213011 0.687223 1.278434 0.883573 1.331544 1.079922 1.371831 1.276272 1.398907 1.472622 1.412510 1.999598 2.000000 2.000000 Rates of convergence: ***T*** **diffs** **ratios** 1.896119 1.974232 0.078113 1.993570 0.019339 0.247575 1.998393 0.004823 0.249397 1.999598 0.001205 0.249849 1.999900 0.000301 0.249962 approx ¼	M1A1 M1A1 M1A1 M1A1 M1A1	Correct table of values used T values T^* T^{**}	

Question			Answer	Marks	Guidance	
3	(ii) cont		**T*** **diifs** **ratios** 2.000269 2.000017 –0.000253 2.000001 –0.000016 0.061595 2.000000 –0.000001 0.062274 2.000000 0.000000 0.062444 approx 1/16 **T**** **diifs** **ratios** 2.000000 2.000000 0.000000 2.000000 0.000000 0.015385 2.000000 0.000000 0.015565 approx 1/64	A1 A1 **[12]**		
3	(iii)		Obtain values of I as follows k I 1 2 0.8 1.870992 0.6 1.731752 0.4 1.579036 0.2 1.40692 0 1.196? Comment that as k gets close to zero convergence is slower / results are less accurate. Comment that ratios of differences drift away from their theoretical values.	M1 A1A1 A1 E1 E1 **[6]**	Spreadsheet handles variable k. Some, all values of I To appropriate accuracy for working shown	

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th colspan="2">Guidance</th></tr>
<tr><td>4</td><td>(i)</td><td></td><td>Diagonal dominance: $a \geq 2$ and $b \geq 4$
Strict diagonal dominance: as above but with at least one ‘>’
Diagonal dominance is not sufficient, strict diagonal dominance is a sufficient but not necessary condition for G-S to converge</td><td>B1B1
B1B1
B1B1

[6]</td><td>B1 inequalities, B1 ‘and’ soi
B1 for >, B1 for correct statement</td><td></td></tr>
<tr><td>4</td><td>(ii)</td><td></td><td>G-S starting, e.g.
<table>
<tr><th>a</th><th>b</th><th>x_1</th><th>x_2</th><th>x_3</th><th>x_4</th></tr>
<tr><td>2</td><td>4</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td></td><td></td><td>2</td><td>–0.25</td><td>0.625</td><td>–0.1875</td></tr>
<tr><td></td><td></td><td>2.03125</td><td>–0.57813</td><td>0.882813</td><td>–0.07422</td></tr>
<tr><td></td><td></td><td>2.251953</td><td>–0.81738</td><td>0.945801</td><td>–0.15308</td></tr>
<tr><td></td><td></td><td>2.332153</td><td>–0.88898</td><td>1.021027</td><td>–0.15556</td></tr>
<tr><td></td><td></td><td>2.366707</td><td>–0.94387</td><td>1.049715</td><td>–0.1585</td></tr>
</table>
and converging to
2.416667 –1 1.083333 –0.16667
Conjecture solution is: 29/12, –1, 13/12, –1/6
Demonstrate correct by substitution</td><td>M1A1

M1A1

M1A1
M1A1
M1A1
[10]</td><td>G-S starts off correctly ... (NB: not G-J)
... continues correctly ...
... converges correctly</td><td></td></tr>
<tr><td>4</td><td>(iii)</td><td></td><td>Examples of fast convergence with a >>2, $b = 4$
Examples of fast convergence with $a = 2$ and b >> 4</td><td>M1A1
M1A1
[4]</td><td></td><td></td></tr>
<tr><td>4</td><td>(iv)</td><td></td><td>Evidence of sensible trial and improvement
Conclude $b = 2.5$ (ie $b = 2.6$ G-S converges)</td><td>M1
A1
[2]</td><td></td><td></td></tr>
<tr><td>4</td><td>(v)</td><td></td><td>Evidence of sensible trial and improvement
Conclude $a = 1.6$ (ie, $a = 1.7$ G-S converges)</td><td>M1
A1
[2]</td><td></td><td></td></tr>
</table>

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

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