



Wednesday 18 June 2014 – Afternoon

A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER



Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Express $\frac{3x}{(2-x)(4+x^2)}$ in partial fractions. [5]

2 Find the first three terms in the binomial expansion of $(4+x)^{\frac{3}{2}}$. State the set of values of x for which the expansion is valid. [5]

3 Fig. 3 shows the curve $y = x^3 + \sqrt{(\sin x)}$ for $0 \leq x \leq \frac{\pi}{4}$.

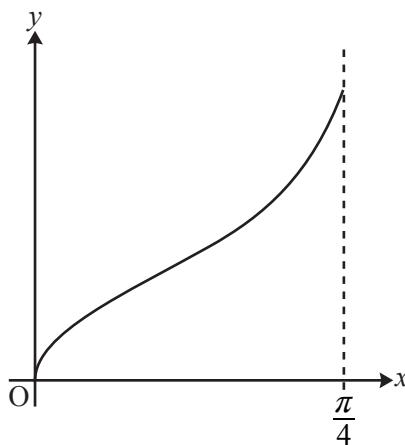


Fig. 3

(i) Use the trapezium rule with 4 strips to estimate the area of the region bounded by the curve, the x -axis and the line $x = \frac{\pi}{4}$, giving your answer to 3 decimal places. [4]

(ii) Suppose the number of strips in the trapezium rule is increased. Without doing further calculations, state, with a reason, whether the area estimate increases, decreases, or it is not possible to say. [1]

4 (i) Show that $\cos(\alpha + \beta) = \frac{1 - \tan \alpha \tan \beta}{\sec \alpha \sec \beta}$. [3]

(ii) Hence show that $\cos 2\alpha = \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha}$. [2]

(iii) Hence or otherwise solve the equation $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \frac{1}{2}$ for $0^\circ \leq \theta \leq 180^\circ$. [3]

5 A curve has parametric equations $x = e^{3t}$, $y = te^{2t}$.

(i) Find $\frac{dy}{dx}$ in terms of t . Hence find the exact gradient of the curve at the point with parameter $t = 1$. [4]

(ii) Find the cartesian equation of the curve in the form $y = ax^b \ln x$, where a and b are constants to be determined. [3]

6 Fig. 6 shows the region enclosed by the curve $y = (1 + 2x^2)^{\frac{1}{3}}$ and the line $y = 2$.

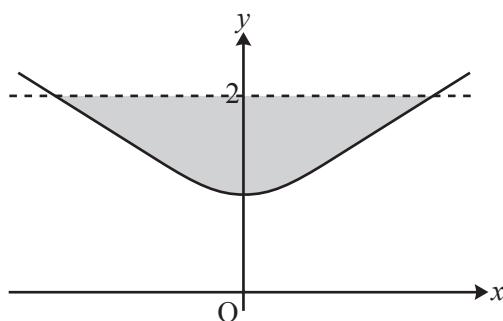


Fig. 6

This region is rotated about the y -axis. Find the volume of revolution formed, giving your answer as a multiple of π . [6]

Question 7 begins on page 4.

Section B (36 marks)

7 Fig. 7 shows a tetrahedron ABCD. The coordinates of the vertices, with respect to axes Oxyz, are A(-3, 0, 0), B(2, 0, -2), C(0, 4, 0) and D(0, 4, 5).

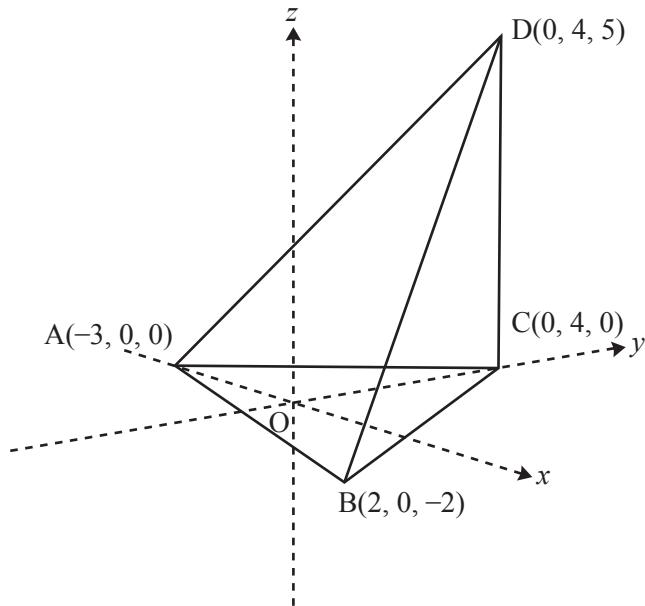


Fig. 7

(i) Find the lengths of the edges AB and AC, and the size of the angle CAB. Hence calculate the area of triangle ABC. [7]

(ii) (A) Verify that $4\mathbf{i} - 3\mathbf{j} + 10\mathbf{k}$ is normal to the plane ABC. [2]

(B) Hence find the equation of this plane. [2]

(iii) Write down a vector equation for the line through D perpendicular to the plane ABC. Hence find the point of intersection of this line with the plane ABC. [5]

The volume of a tetrahedron is $\frac{1}{3} \times \text{area of base} \times \text{height}$.

(iv) Find the volume of the tetrahedron ABCD. [2]

8 Fig. 8.1 shows an upright cylindrical barrel containing water. The water is leaking out of a hole in the side of the barrel.

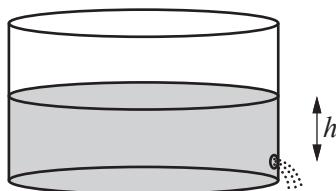


Fig. 8.1

The height of the water surface above the hole t seconds after opening the hole is h metres, where

$$\frac{dh}{dt} = -A\sqrt{h}$$

and where A is a positive constant. Initially the water surface is 1 metre above the hole.

(i) Verify that the solution to this differential equation is

$$h = \left(1 - \frac{1}{2}At\right)^2. \quad [3]$$

The water stops leaking when $h = 0$. This occurs after 20 seconds.

(ii) Find the value of A , and the time when the height of the water surface above the hole is 0.5 m. [4]

Fig. 8.2 shows a similar situation with a different barrel; h is in metres.

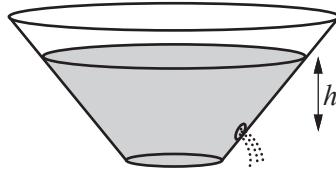


Fig. 8.2

For this barrel,

$$\frac{dh}{dt} = -B \frac{\sqrt{h}}{(1+h)^2},$$

where B is a positive constant. When $t = 0$, $h = 1$.

(iii) Solve this differential equation, and hence show that

$$h^{\frac{1}{2}}(30 + 20h + 6h^2) = 56 - 15Bt. \quad [7]$$

(iv) Given that $h = 0$ when $t = 20$, find B .

Find also the time when the height of the water surface above the hole is 0.5 m. [4]

END OF QUESTION PAPER

BLANK PAGE



Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.