



## Monday 23 June 2014 – Morning

### A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

#### QUESTION PAPER



Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

#### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

#### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

#### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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## Section A (54 marks)

1 (a) Given that  $f(x) = \arccos x$ ,

(i) sketch the graph of  $y = f(x)$ , [2]

(ii) show that  $f'(x) = -\frac{1}{\sqrt{1-x^2}}$ , [3]

(iii) obtain the Maclaurin series for  $f(x)$  as far as the term in  $x^3$ . [7]

(b) A curve has polar equation  $r = \theta + \sin \theta, \theta \geq 0$ .

(i) By considering  $\frac{dr}{d\theta}$  show that  $r$  increases as  $\theta$  increases.

Sketch the curve for  $0 \leq \theta \leq 4\pi$ . [4]

(ii) You are given that  $\sin \theta \approx \theta$  for small  $\theta$ . Find in terms of  $\alpha$  the approximate area bounded by the curve and the lines  $\theta = 0$  and  $\theta = \alpha$ , where  $\alpha$  is small. [3]

2 (a) The infinite series  $C$  and  $S$  are defined as follows.

$$C = a \cos \theta + a^2 \cos 2\theta + a^3 \cos 3\theta + \dots,$$

$$S = a \sin \theta + a^2 \sin 2\theta + a^3 \sin 3\theta + \dots,$$

where  $a$  is a real number and  $|a| < 1$ .

By considering  $C + jS$ , show that

$$S = \frac{a \sin \theta}{1 - 2a \cos \theta + a^2}.$$

Find a corresponding expression for  $C$ . [8]

(b)  $P$  is one vertex of a regular hexagon in an Argand diagram. The centre of the hexagon is at the origin.  $P$  corresponds to the complex number  $\sqrt{3} + j$ .

(i) Find, in the form  $x + jy$ , the complex numbers corresponding to the other vertices of the hexagon. [5]

(ii) The six complex numbers corresponding to the vertices of the hexagon are squared to form the vertices of a new figure. Find, in the form  $x + jy$ , the vertices of the new figure. Find the area of the new figure. [4]

3 (a) (i) Find the eigenvalues and corresponding eigenvectors for the matrix  $\mathbf{A}$ , where

$$\mathbf{A} = \begin{pmatrix} 6 & -3 \\ 4 & -1 \end{pmatrix}.$$

[5]

(ii) Write down a matrix  $\mathbf{P}$  and a diagonal matrix  $\mathbf{D}$  such that  $\mathbf{A} = \mathbf{PDP}^{-1}$ .

[2]

(b) (i) The  $3 \times 3$  matrix  $\mathbf{B}$  has characteristic equation

$$\lambda^3 - 4\lambda^2 - 3\lambda - 10 = 0.$$

Show that 5 is an eigenvalue of  $\mathbf{B}$ . Show that  $\mathbf{B}$  has no other real eigenvalues.

[4]

(ii) An eigenvector corresponding to the eigenvalue 5 is  $\begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$ .

Evaluate  $\mathbf{B} \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$  and  $\mathbf{B}^2 \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$ .

Solve the equation  $\mathbf{B} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -20 \\ 10 \\ 40 \end{pmatrix}$  for  $x, y, z$ .

[4]

(iii) Show that  $\mathbf{B}^4 = 19 \mathbf{B}^2 + 22 \mathbf{B} + 40 \mathbf{I}$ .

[3]

### Section B (18 marks)

4 (i) Given that  $\sinh y = x$ , show that

$$y = \ln(x + \sqrt{1+x^2}). \quad (*)$$

Differentiate (\*) to show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}.$$

[8]

(ii) Find  $\int \frac{1}{\sqrt{25+4x^2}} dx$ , expressing your answer in logarithmic form.

[3]

(iii) Use integration by substitution with  $2x = 5 \sinh u$  to show that

$$\int \sqrt{25+4x^2} dx = \frac{25}{4} \left( \ln \left( \frac{2x}{5} + \sqrt{1 + \frac{4x^2}{25}} \right) + \frac{2x}{5} \sqrt{1 + \frac{4x^2}{25}} \right) + c,$$

where  $c$  is an arbitrary constant.

[7]

**END OF QUESTION PAPER**



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