Monday 23 June 2014 – Morning

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (54 marks)

1 **(a)** Given that $f(x) = \arccos x$,

(i) sketch the graph of $y = f(x)$, **[2]**

(ii) show that $f'(x) = -\dfrac{1}{\sqrt{1-x^2}}$, **[3]**

(iii) obtain the Maclaurin series for $f(x)$ as far as the term in x^3. **[7]**

(b) A curve has polar equation $r = \theta + \sin\theta$, $\theta \geqslant 0$.

(i) By considering $\dfrac{dr}{d\theta}$ show that r increases as θ increases.

Sketch the curve for $0 \leqslant \theta \leqslant 4\pi$. **[4]**

(ii) You are given that $\sin\theta \approx \theta$ for small θ. Find in terms of α the approximate area bounded by the curve and the lines $\theta = 0$ and $\theta = \alpha$, where α is small. **[3]**

2 **(a)** The infinite series C and S are defined as follows.

$$C = a\cos\theta + a^2\cos 2\theta + a^3\cos 3\theta + \ldots,$$

$$S = a\sin\theta + a^2\sin 2\theta + a^3\sin 3\theta + \ldots,$$

where a is a real number and $|a| < 1$.

By considering $C + jS$, show that

$$S = \frac{a\sin\theta}{1 - 2a\cos\theta + a^2}.$$

Find a corresponding expression for C. **[8]**

(b) P is one vertex of a regular hexagon in an Argand diagram. The centre of the hexagon is at the origin. P corresponds to the complex number $\sqrt{3} + j$.

(i) Find, in the form $x + jy$, the complex numbers corresponding to the other vertices of the hexagon. **[5]**

(ii) The six complex numbers corresponding to the vertices of the hexagon are squared to form the vertices of a new figure. Find, in the form $x + jy$, the vertices of the new figure. Find the area of the new figure. **[4]**

3 **(a)** **(i)** Find the eigenvalues and corresponding eigenvectors for the matrix **A**, where

$$\mathbf{A} = \begin{pmatrix} 6 & -3 \\ 4 & -1 \end{pmatrix}.$$ **[5]**

(ii) Write down a matrix **P** and a diagonal matrix **D** such that $\mathbf{A} = \mathbf{PDP}^{-1}$. **[2]**

(b) **(i)** The 3×3 matrix **B** has characteristic equation

$$\lambda^3 - 4\lambda^2 - 3\lambda - 10 = 0.$$

Show that 5 is an eigenvalue of **B**. Show that **B** has no other real eigenvalues. **[4]**

(ii) An eigenvector corresponding to the eigenvalue 5 is $\begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$.

Evaluate $\mathbf{B}\begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$ and $\mathbf{B}^2\begin{pmatrix} 4 \\ -2 \\ -8 \end{pmatrix}$.

Solve the equation $\mathbf{B}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -20 \\ 10 \\ 40 \end{pmatrix}$ for x, y, z. **[4]**

(iii) Show that $\mathbf{B}^4 = 19\,\mathbf{B}^2 + 22\,\mathbf{B} + 40\,\mathbf{I}$. **[3]**

Section B (18 marks)

4 **(i)** Given that $\sinh y = x$, show that

$$y = \ln\left(x + \sqrt{1 + x^2}\right). \qquad (*)$$

Differentiate (*) to show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 + x^2}}.$$ **[8]**

(ii) Find $\int \frac{1}{\sqrt{25 + 4x^2}}\,dx$, expressing your answer in logarithmic form. **[3]**

(iii) Use integration by substitution with $2x = 5\sinh u$ to show that

$$\int \sqrt{25 + 4x^2}\,dx = \frac{25}{4}\left(\ln\left(\frac{2x}{5} + \sqrt{1 + \frac{4x^2}{25}}\right) + \frac{2x}{5}\sqrt{1 + \frac{4x^2}{25}}\right) + c,$$

where c is an arbitrary constant. **[7]**

END OF QUESTION PAPER