



**Friday 6 June 2014 – Afternoon**

**A2 GCE MATHEMATICS (MEI)**

**4757/01** Further Applications of Advanced Mathematics (FP3)

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4757/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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*Option 1: Vectors*

- 1 Three points have coordinates  $A(-3, 12, -7)$ ,  $B(-2, 6, 9)$ ,  $C(6, 0, -10)$ . The plane  $P$  passes through the points  $A$ ,  $B$  and  $C$ .

(i) Find the vector product  $\overrightarrow{AB} \times \overrightarrow{AC}$ . Hence or otherwise find an equation for the plane  $P$  in the form  $ax + by + cz = d$ . [5]

The plane  $Q$  has equation  $6x + 3y + 2z = 32$ . The perpendicular from  $A$  to the plane  $Q$  meets  $Q$  at the point  $D$ . The planes  $P$  and  $Q$  intersect in the line  $L$ .

(ii) Find the distance  $AD$ . [3]

(iii) Find an equation for the line  $L$ . [5]

(iv) Find the shortest distance from  $A$  to the line  $L$ . [6]

(v) Find the volume of the tetrahedron  $ABCD$ . [5]

*Option 2: Multi-variable calculus*

- 2 A surface  $S$  has equation  $g(x, y, z) = 0$ , where  $g(x, y, z) = x^2 + 3y^2 + 2z^2 + 2yz + 6xz - 4xy - 24$ .  $P(2, 6, -2)$  is a point on the surface  $S$ .

(i) Find  $\frac{\partial g}{\partial x}$ ,  $\frac{\partial g}{\partial y}$  and  $\frac{\partial g}{\partial z}$ . [3]

(ii) Find the equation of the normal line to the surface  $S$  at the point  $P$ . [3]

(iii) The point  $Q$  is on this normal line and close to  $P$ . At  $Q$ ,  $g(x, y, z) = h$ , where  $h$  is small. Find, in terms of  $h$ , the approximate perpendicular distance from  $Q$  to the surface  $S$ . [4]

(iv) Find the coordinates of the two points on the surface at which the normal line is parallel to the  $y$ -axis. [6]

(v) Given that  $10x - y + 2z = 6$  is the equation of a tangent plane to the surface  $S$ , find the coordinates of the point of contact. [8]

## Option 3: Differential geometry

- 3 (a) A curve has intrinsic equation  $s = 2 \ln \left( \frac{\pi}{\pi - 3\psi} \right)$  for  $0 \leq \psi < \frac{1}{3}\pi$ , where  $s$  is the arc length measured from a fixed point P and  $\tan \psi = \frac{dy}{dx}$ . P is in the third quadrant. The curve passes through the origin O, at which point  $\psi = \frac{1}{6}\pi$ . Q is the point on the curve at which  $\psi = \frac{3}{10}\pi$ .
- (i) Express  $\psi$  in terms of  $s$ , and sketch the curve, indicating the points O, P and Q. [4]
  - (ii) Find the arc length OQ. [3]
  - (iii) Find the radius of curvature at the point O. [3]
  - (iv) Find the coordinates of the centre of curvature corresponding to the point O. [3]
- (b) (i) Find the surface area of revolution formed when the curve  $y = \frac{1}{3}\sqrt{x}(x-3)$  for  $1 \leq x \leq 4$  is rotated through  $2\pi$  radians about the  $y$ -axis. [7]
- (ii) The curve in part (b)(i) is one member of the family  $y = \frac{1}{9}\lambda\sqrt{x}(x-\lambda)$ , where  $\lambda$  is a positive parameter. Find the equation of the envelope of this family of curves. [4]

## Option 4: Groups

- 4 The twelve distinct elements of an abelian multiplicative group  $G$  are

$$e, a, a^2, a^3, a^4, a^5, b, ab, a^2b, a^3b, a^4b, a^5b$$

where  $e$  is the identity element,  $a^6 = e$  and  $b^2 = e$ .

- (i) Show that the element  $a^2b$  has order 6. [3]
- (ii) Show that  $\{e, a^3, b, a^3b\}$  is a subgroup of  $G$ . [3]
- (iii) List all the cyclic subgroups of  $G$ . [6]

You are given that the set

$$H = \{1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61, 67, 71, 73, 77, 79, 83, 89\}$$

with binary operation multiplication modulo 90 is a group.

- (iv) Determine the order of each of the elements 11, 17 and 19. [4]
- (v) Give a cyclic subgroup of  $H$  with order 4. [2]
- (vi) By identifying possible values for the elements  $a$  and  $b$  above, or otherwise, give one example of each of the following:
  - (A) a non-cyclic subgroup of  $H$  with order 12, [3]
  - (B) a non-cyclic subgroup of  $H$  with order 4. [3]

## Option 5: Markov chains

**This question requires the use of a calculator with the ability to handle matrices.**

**5** In this question, give probabilities correct to 4 decimal places.

The speeds of vehicles are measured on a busy stretch of road and are categorised as A (not more than 30 mph), B (more than 30 mph but not more than 40 mph) or C (more than 40 mph).

- Following a vehicle in category A, the probabilities that the next vehicle is in categories A, B, C are 0.9, 0.07, 0.03 respectively.
- Following a vehicle in category B, the probabilities that the next vehicle is in categories A, B, C are 0.3, 0.6, 0.1 respectively.
- Following a vehicle in category C, the probabilities that the next vehicle is in categories A, B, C are 0.1, 0.7, 0.2 respectively.

This is modelled as a Markov chain with three states corresponding to the categories A, B, C. The speed of the first vehicle is measured as 28 mph.

- Write down the transition matrix  $\mathbf{P}$ . [2]
- Find the probabilities that the 10th vehicle is in each of the three categories. [3]
- Find the probability that the 12th and 13th vehicles are in the same category. [4]
- Find the smallest value of  $n$  for which the probability that the  $n$ th and  $(n+1)$ th vehicles are in the same category is less than 0.8, and give the value of this probability. [4]
- Find the expected number of vehicles (including the first vehicle) in category A before a vehicle in a different category. [2]
- Find the limit of  $\mathbf{P}^n$  as  $n$  tends to infinity, and hence write down the equilibrium probabilities for the three categories. [3]
- Find the probability that, after many vehicles have passed by, the next three vehicles are all in category A. [2]

On a new stretch of road, the same categories are used but some of the transition probabilities are different.

- Following a vehicle in category A, the probability that the next vehicle is in category B is equal to the probability that it is in category C.
- Following a vehicle in category B, the probability that the next vehicle is in category A is equal to the probability that it is in category C.
- Following a vehicle in category C, the probabilities that the next vehicle is in categories A, B, C are 0.1, 0.7, 0.2 respectively.

In the long run, the proportions of vehicles in categories A, B, C are 50%, 40%, 10% respectively.

- Find the transition matrix for the new stretch of road. [4]

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