OCR

Thursday 12 June 2014 – Afternoon

A2 GCE MATHEMATICS (MEI)

4768/01 Statistics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4768/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 **(i)** Let X be a random variable with variance σ^2. The independent random variables X_1 and X_2 are both distributed as X. Write down the variances of $X_1 + X_2$ and $2X$; explain why they are different. **[3]**

A large company has produced an aptitude test which consists of three parts. The parts are called mathematical ability, spatial awareness and communication. The scores obtained by candidates in the three parts are continuous random variables X, Y and W which have been found to have independent Normal distributions with means and standard deviations as shown in the table.

	Mean	Standard deviation
Mathematical ability, X	30.1	5.1
Spatial awareness, Y	25.4	4.2
Communication, W	28.2	3.9

(ii) Find the probability that a randomly selected candidate obtains a score of less than 22 in the mathematical ability part of the test. **[3]**

(iii) Find the probability that a randomly selected candidate obtains a total score of at least 100 in the whole test. **[4]**

(iv) For a particular role in the company, the score $2X + Y$ is calculated. Find the score that is exceeded by only 2% of candidates. **[4]**

(v) For a different role, a candidate must achieve a score in communication which is at least 60% of the score obtained in mathematical ability. What proportion of candidates do not achieve this? **[3]**

2 **(i)** Explain what is meant by a simple random sample. **[2]**

A manufacturer produces tins of paint which nominally contain 1 litre. The quantity of paint delivered by the machine that fills the tins can be assumed to be a Normally distributed random variable.

The machine is designed to deliver an average of 1.05 litres to each tin. However, over time paint builds up in the delivery nozzle of the machine, reducing the quantity of paint delivered. Random samples of 10 tins are taken regularly from the production process. If a significance test, carried out at the 5% level, suggests that the average quantity of paint delivered is less than 1.02 litres, the machine is cleaned.

(ii) By carrying out an appropriate test, determine whether or not the sample below leads to the machine being cleaned.

0.994 1.010 1.021 1.015 1.016 1.022 1.009 1.007 1.011 1.026 **[9]**

Each time the machine has been cleaned, a random sample of 10 tins is taken to determine whether or not the average quantity of paint delivered has returned to 1.05 litres.

(iii) On one occasion after the machine has been cleaned, the quality control manager thinks that the distribution of the quantity of paint is symmetrical but not necessarily Normal. The sample on this occasion is as follows.

1.055 1.064 1.063 1.043 1.062 1.070 1.059 1.044 1.054 1.053

By carrying out an appropriate test at the 5% level of significance, determine whether or not this sample supports the conclusion that the average quantity of paint delivered is 1.05 litres. **[8]**

3 **(a)** A personal trainer believes that drinking a glass of beetroot juice an hour before exercising enables endurance tests to be completed more quickly. To test his belief he takes a random sample of 12 of his trainees and, on two occasions, asks them to carry out 100 repetitions of a particular exercise as quickly as possible. Each trainee drinks a glass of water on one occasion and a glass of beetroot juice on the other occasion.

The times in seconds taken by the trainees are given in the table.

Trainee	Water	Beetroot juice
A	75.1	72.9
B	86.2	79.9
C	77.3	71.6
D	89.1	90.2
E	67.9	68.2
F	101.5	95.2
G	82.5	76.5
H	83.3	80.2
I	102.5	99.1
J	91.3	82.2
K	92.5	90.1
L	77.2	77.9

The trainer wishes to test his belief using a paired t test at the 1% level of significance. Assuming any necessary assumptions are valid, carry out a test of the hypotheses $H_0: \mu_D = 0$, $H_1: \mu_D < 0$, where μ_D is the population mean difference in times (time with beetroot juice minus time with water). **[8]**

(b) An ornithologist believes that the number of birds landing on the bird feeding station in her garden in a given interval of time during the morning should follow a Poisson distribution. In order to test her belief, she makes the following observations in 60 randomly chosen minutes one morning.

Number of birds	0	1	2	3	4	5	6	$\geqslant 7$
Frequency	2	5	10	17	14	7	4	1

Given that the data in the table have a mean value of 3.3, use a goodness of fit test, with a significance level of 5%, to investigate whether the ornithologist is justified in her belief. **[11]**

Question 4 begins on page 4

4 The probability density function of a random variable X is given by

$$f(x) = \begin{cases} kx & 0 \leqslant x \leqslant a, \\ k(2a-x) & a < x \leqslant 2a, \\ 0 & \text{otherwise,} \end{cases}$$

where a and k are positive constants.

(i) Sketch $f(x)$. Hence explain why $E(X) = a$. **[3]**

(ii) Show that $k = \frac{1}{a^2}$. **[3]**

(iii) Find $\text{Var}(X)$ in terms of a. **[4]**

In order to estimate the value of a, a random sample of size 50 is taken from the distribution. It is found that the sample mean and standard deviation are $\overline{x} = 1.92$ and $s = 0.8352$.

(iv) Construct a symmetrical 95% confidence interval for a. Give one reason why the answer is only approximate. **[5]**

(v) A non-statistician states that the probability that a lies in the interval found in part **(iv)** is 0.95. Comment on this statement. **[2]**

END OF QUESTION PAPER