



**Monday 16 June 2014 – Morning**

**AS GCE MATHEMATICS (MEI)**

**4776/01** Numerical Methods

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4776/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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**Section A** (36 marks)

- 1** You are given that the equation

$$x^2 + \frac{1}{x} - 5 = 0$$

has one root,  $\alpha$ , in the interval  $(0.2, 0.3)$  and another root,  $\beta$ , in the interval  $(2, 2.2)$ .

- (i) Use the method of false position with starting values 0.2 and 0.3 to find a first estimate of  $\alpha$ . Apply false position again to find a second estimate of  $\alpha$ .

A third application of the false position method gives 0.201 640 as an estimate of  $\alpha$ . Give the value of  $\alpha$  to the accuracy that appears to be justified. [5]

- (ii) Suppose that  $\beta$  is to be found using the bisection method, starting with the interval  $(2, 2.2)$ .

Write down the initial estimate of  $\beta$  and the associated maximum possible error.

Determine how many further steps of the bisection method would be required to find  $\beta$  with a maximum possible error of less than 0.005. (You are not required to carry out this bisection process.) [3]

- 2** The table shows several values of the mid-point rule ( $M$ ) and trapezium rule ( $T$ ) estimates of an integral for different values of  $h$ .

$h$	$M$	$T$
0.5	0.536 650	0.594 027
0.25		
0.125	0.555 282	0.558 599

- (i) Calculate the missing values in the table. Calculate also three Simpson's rule estimates of the integral. [7]

- (ii) Give the value of the integral to the accuracy that appears justified. [1]

- 3** In computer science, powers of 2 occur frequently. It is quite common to approximate  $2^{10k}$  as  $10^{3k}$ .

- (i) Find the relative error in this approximation for each of  $k = 1$ ,  $k = 2$  and  $k = 3$ . [4]

- (ii) You are reminded that if  $X = x(1 + r)$ , where  $r$  is small, then  $X^n \approx x^n(1 + nr)$ .

How does this result relate to your answers in part (i)? [2]

- 4 The function  $f(x)$  has the values shown in the table.

$x$	1.9	1.95	2	2.05	2.1
$f(x)$	0.385 570	0.677 625	1	1.356 076	1.749 638

- (i) Calculate two estimates of  $f'(2)$  using the forward difference method. [2]
- (ii) Calculate two estimates of  $f'(2)$  using the central difference method. [2]
- (iii) Give the value of  $f'(2)$  to the accuracy that appears justified, explaining your reasoning. [2]

- 5 The table gives some values of the function  $g(x)$  correct to 6 decimal places.

$x$	1	1.1	1.2
$g(x)$	1.188 395	1.234 281	1.287 500

- (i) Estimate  $g(1.14)$  using linear interpolation. [2]
- (ii) Use Newton's forward difference interpolation formula to write down a quadratic approximation to  $g(x)$ . You need not simplify your answer. [4]
- (iii) Hence obtain a second estimate of  $g(1.14)$ . [2]

### Section B (36 marks)

- 6 You are given that the equation

$$6x^5 - 3x^2 - 2x + 1 = 0$$

has exactly 3 real roots. Let these roots be  $\alpha, \beta, \gamma$  where  $\alpha < \beta < \gamma$ .

- (i) Show that there is one root in the interval  $(-1, 0)$  and two roots in the interval  $(0, 1)$ . [3]
- (ii) Use the iteration

$$x_{r+1} = \left( \frac{1}{6} (3x_r^2 + 2x_r - 1) \right)^{\frac{1}{5}}$$

to find  $\alpha$  correct to 2 decimal places. [4]

- (iii) Use the iteration

$$x_{r+1} = \frac{1}{2} (6x_r^5 - 3x_r^2 + 1) \quad (*)$$

to verify that  $\beta \approx 0.34$ .

By considering a suitable derivative, show that the convergence of  $(*)$  to  $\beta$  will be slow. [6]

- (iv) Use the Newton-Raphson method to find  $\gamma$  correct to 4 decimal places. [5]

- 7 A function  $f(x)$  is known to have the following values correct to 3 decimal places.

$x$	0	0.5	1	1.5
$f(x)$	1.693	1.405	1.288	1.253

The value of the integral  $I = \int_0^{1.5} f(x) dx$  is required.

You should give all numerical answers in this question correct to 5 decimal places.

- (i) Find the best estimate possible of  $I$  using just the trapezium rule. [2]
- (ii) By drawing a suitable graph, determine whether your answer in part (i) is likely to be an underestimate or an overestimate. [4]
- (iii) Obtain two further estimates of  $I$ , each of them using both the trapezium rule and Simpson's rule. With reference to your graph, explain which of these estimates seems likely to be more accurate. [6]
- (iv) The integration rule

$$\int_0^{3h} f(x) dx \approx \frac{3h}{8}(f(0) + 3f(h) + 3f(2h) + f(3h))$$

is designed to be used when 4 equally spaced values of a function are known.

Find the value of  $I$  given by this rule.

You should now assume that the value of  $I$  you have just calculated is correct to 3 decimal places. Find the approximate errors in your three earlier estimates of  $I$ , and hence determine whether your conclusions in parts (ii) and (iii) are confirmed. [6]

### END OF QUESTION PAPER



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