OCR

Tuesday 24 June 2014 – Morning

A2 GCE MATHEMATICS (MEI)

4798/01 Further Pure Mathematics with Technology (FPT)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4798/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software

Duration: Up to 2 hours

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a computer algebra system, a spreadsheet, a programming language and graph-plotting software throughout the examination.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 This question concerns curves with equation

$$y = \frac{x^3 + ax^2 + 1}{x^n}$$

for various values of a and n.

(i) For the case $n = 1$, sketch the curves when $a = 2$, $a = 3$ and $a = 4$. Describe two common features of these three curves. **[6]**

(ii) For the case $n = 1$ and $a = 3$, find the number of stationary points and identify their nature, justifying your answers. **[5]**

(iii) For the case $a = 3$, sketch the curves when $n = 2$ and $n = 3$. **[4]**

(iv) Given that $a > 0$, find the equations of the asymptotes for each of the cases $n = 2$ and $n = 3$. For any non-vertical asymptotes, state whether they are approached from above or below, justifying your answers. **[8]**

2 **(i)** The function f is defined by $\mathrm{f}(z) = z^3 + (2-2\mathrm{i})z^2 + (7-12\mathrm{i})z + 6 - 10\mathrm{i}$. Solve the equation $\mathrm{f}(z) = 0$ and plot the roots as points on an Argand diagram.

Show that these points lie on a straight line. **[6]**

(ii) Find the roots of $\mathrm{f}'(z) = 0$. Plot these roots on the Argand diagram drawn in part **(i)**.

Show that the roots of $\mathrm{f}'(z) = 0$ lie on the same straight line as the roots of $\mathrm{f}(z) = 0$. **[6]**

(iii) The function g is defined by $\mathrm{g}(z) = z^3 - (k+1)az^2 + ka^2z$ where $a \in \mathbb{C}$, $k \in \mathbb{R}$.

Show that the roots of $\mathrm{g}(z) = 0$ lie on a straight line.

Show that the roots of $\mathrm{g}'(z) = 0$ lie on this same line. **[8]**

(iv) Now consider a function h which is a cubic with real coefficients. Identify the two distinct conditions under which the roots of $\mathrm{h}(z) = 0$ lie on a straight line in the Argand diagram. Give, in expanded form, an example of such a cubic for each case. **[5]**

3 This question concerns Pythagorean triples: positive integers a, b and c such that $a^2 + b^2 = c^2$. The integer n is defined by $c = b + n$.

(i) Create a program that will find all such triples for a given value of n, where both a and b are less than or equal to a maximum value, m. You should write out your program in full.

For the case $n = 1$, find all the triples with $1 \leqslant a \leqslant 100$ and $1 \leqslant b \leqslant 100$.

For the case $n = 3$, find all the triples with $1 \leqslant a \leqslant 200$ and $1 \leqslant b \leqslant 200$. **[9]**

(ii) For the case $n = 1$, prove that there is a triple for every odd value of a where $a > 1$. **[4]**

(iii) For the case $n = p$, where p is prime, show that a must be a multiple of p. **[3]**

(iv) For the case $n = b$, determine whether there are any triples. **[4]**

(v) Edit your program from part **(i)** so that it will only find values of a and b where b is not a multiple of n. Indicate clearly all the changes to your program.

Use the edited program to find all such triples for the case $n = 2$ with $1 \leqslant a \leqslant 100$ and $1 \leqslant b \leqslant 100$. **[4]**

END OF QUESTION PAPER