

OXFORD CAMBRIDGE AND RSA EXAMINATIONS
A2 GCE
4758/01

MATHEMATICS (MEI)
Differential Equations
QUESTION PAPER

THURSDAY 12 JUNE 2014: Afternoon
DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4758/01
MEI Examination Formulae and Tables (MF2)
Insert for question 3(b)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK.

Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer any **THREE** questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive **NO MARKS** unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is **72**.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

- 1 The displacement, x m, of a particle at time t s is given by the differential equation**

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 30 \cos 2t.$$

- (i) Find the general solution. [9]**

The particle is initially at the origin, travelling with velocity 10 m s^{-1} .

- (ii) Find the particular solution. [4]**

- (iii) Find the amplitude of the oscillations of the particle for large values of t . [2]**

Consider now the differential equation

$$\frac{d^3x}{dt^3} + 2\frac{d^2x}{dt^2} + 2\frac{dx}{dt} = 0.$$

- (iv) Show that 0 is a root of the auxiliary equation and write down the other roots. [2]**

- (v) Find the particular solution of this differential equation subject to the initial conditions**

$$x = 0, \frac{dx}{dt} = 10 \text{ and } \frac{d^2x}{dt^2} = 4 \text{ when } t = 0. \quad [5]$$

- (vi) Sketch the graph of this solution. [2]**

- 2 The population, P , of a species at time t hours is to be modelled by a differential equation. The initial population is 100.**

At first, the model $\frac{dP}{dt} - 0.25P = 0$ is used.

- (i) Find P in terms of t and comment on the suitability of this model. [4]**

To allow for certain environmental effects, the model is refined to

$$\frac{dP}{dt} - 0.25P = -18e^{-0.5t}.$$

- (ii) Write down the complementary function for this differential equation. Find a particular integral and hence state the general solution. [6]**
- (iii) Find the solution subject to the given initial condition and comment on the suitability of this refined model. [3]**

The following mathematical model for the population is now used.

$$\frac{dP}{dt} = 6 \times 10^{-4}P(400 - P)$$

- (iv) Solve this differential equation subject to the given initial condition, expressing P in terms of t . [8]**

- (v) Show that the time T hours at which $P = 200$ is given by

$$T = \frac{25}{6} \ln 3. \quad [1]$$

- (vi) What does this model predict for the population of the species in the long term? [2]

- 3 (a) The equation of a curve in the x - y plane satisfies the differential equation**

$$(x + 1) \frac{dy}{dx} - xy = e^{2x}$$

for $x > -1$.

- (i) Show that an integrating factor for this differential equation is $e^{-x}(1 + x)$ and hence find the general solution for y in terms of x .**

[11]

The curve passes through the point $(0, -2)$.

- (ii) Find the equation of this curve. [2]**

- (b) The differential equation**

$$\frac{dy}{dx} = \frac{1}{x^2 + y^2}$$

is to be solved approximately, first by using a tangent field and then by Euler's method.

- (i) Show that the isocline for which $\frac{dy}{dx} = 4$ is a circle and state its centre and radius. [2]**

- (ii) Sketch the isoclines for the cases**

$$\frac{dy}{dx} = \frac{1}{4}, \frac{dy}{dx} = 1 \text{ and } \frac{dy}{dx} = 4. \text{ Use these isoclines}$$

to draw a tangent field. [3]

- (iii) Sketch the solution curve through $(0, 1)$. [2]**

Euler's method is now used, starting at

$x = 0, y = 1$. The algorithm is given by

$$x_{r+1} = x_r + h, y_{r+1} = y_1 + hy'_r.$$

(iv) Use a step length of 0.05 to estimate y when $x = 0.15$. [4]

4 The simultaneous differential equations

$$\frac{dx}{dt} + 2x = 4y + e^{-2t}$$

$$\frac{dy}{dt} + 3x = 5y + 2e^{-2t}$$

are to be solved.

(i) Obtain a second order differential equation for x in terms of t . Hence find the general solution for x . [12]

(ii) Find the corresponding general solution for y . [4]

When $t = 0$, $y = -\frac{2}{3}$ and $\frac{dy}{dt} = 0$.

(iii) Find the particular solutions for x and y . [5]

(iv) Find the set of values of t for which $y > x$. [3]

END OF QUESTION PAPER

BLANK PAGE

BLANK PAGE



Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

