



**Monday 16 June 2014 – Morning**

**A2 GCE MATHEMATICS (MEI)**

**4753/01** Methods for Advanced Mathematics (C3)

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

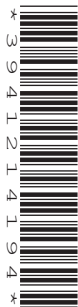
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

**MODIFIED LANGUAGE**



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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## Section A (36 marks)

- 1 Evaluate  $\int_0^{\frac{1}{6}\pi} (1 - \sin 3x) dx$ , giving your answer in exact form. [3]
- 2 Find the exact gradient of the curve  $y = \ln(1 - \cos 2x)$  at the point with  $x$ -coordinate  $\frac{1}{6}\pi$ . [5]
- 3 Solve the equation  $|3 - 2x| = 4|x|$ . [4]

- 4 Fig. 4 shows the curve  $y = f(x)$ , where

$$f(x) = a + \cos bx, 0 \leq x \leq 2\pi,$$

and  $a$  and  $b$  are positive constants. The curve has stationary points at  $(0, 3)$  and  $(2\pi, 1)$ .

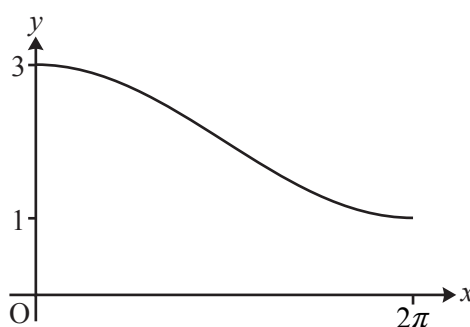


Fig. 4

- (i) Find  $a$  and  $b$ . [2]
  - (ii) Find  $f^{-1}(x)$ , and state its domain and range. [5]
- 5 A spherical balloon of radius  $r$  cm has volume  $V$  cm<sup>3</sup>, where  $V = \frac{4}{3}\pi r^3$ . The balloon is inflated at a constant rate of 10 cm<sup>3</sup> s<sup>-1</sup>. Find the rate of increase of  $r$  when  $r = 8$ . [5]
- 6 The value £ $V$  of a car  $t$  years after it is new is modelled by the equation  $V = Ae^{-kt}$ .  $A$  and  $k$  are positive constants which depend on the make and model of the car.
  - (i) Brian buys a new sports car. Its value is modelled by the equation
 
$$V = 20000e^{-0.2t}.$$
 Calculate how much value, to the nearest £100, this car has lost after 1 year. [2]
  - (ii) Kate buys a new hatchback for £15000 at the same time as Brian buys his car. Her car loses £2000 of its value in the first year. Show that, for Kate's car,  $k = 0.143$  correct to 3 significant figures. [3]
  - (iii) Find how long it is before Brian's and Kate's cars have the same value. [3]
- 7 Either prove or disprove each of the following statements.
  - (i) 'If  $m$  and  $n$  are consecutive odd numbers, then at least one of  $m$  and  $n$  is a prime number.' [2]
  - (ii) 'If  $m$  and  $n$  are consecutive even numbers, then  $mn$  is divisible by 8.' [2]

## Section B (36 marks)

- 8 Fig. 8 shows the curve  $y = f(x)$ , where  $f(x) = \frac{x}{\sqrt{2+x^2}}$ .

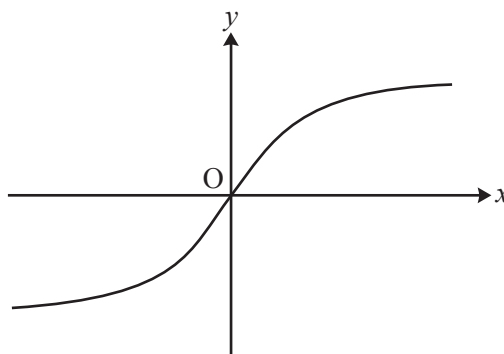


Fig. 8

- (i) Show algebraically that  $f(x)$  is an odd function. Interpret this result geometrically. [3]
- (ii) Show that  $f'(x) = \frac{2}{(2+x^2)^{\frac{3}{2}}}$ . Hence find the exact gradient of the curve at the origin. [5]
- (iii) Find the exact area of the region bounded by the curve, the  $x$ -axis and the line  $x = 1$ . [4]
- (iv) (A) Show that if  $y = \frac{x}{\sqrt{2+x^2}}$ , then  $\frac{1}{y^2} = \frac{2}{x^2} + 1$ . [2]
- (B) Differentiate  $\frac{1}{y^2} = \frac{2}{x^2} + 1$  implicitly to show that  $\frac{dy}{dx} = \frac{2y^3}{x^3}$ . Explain why this expression cannot be used to find the gradient of the curve at the origin. [4]

[Question 9 is printed overleaf.]

- 9 Fig. 9 shows the curve  $y = xe^{-2x}$  together with the straight line  $y = mx$ , where  $m$  is a constant, with  $0 < m < 1$ . The curve and the line meet at O and P. The dashed line is the tangent at P.

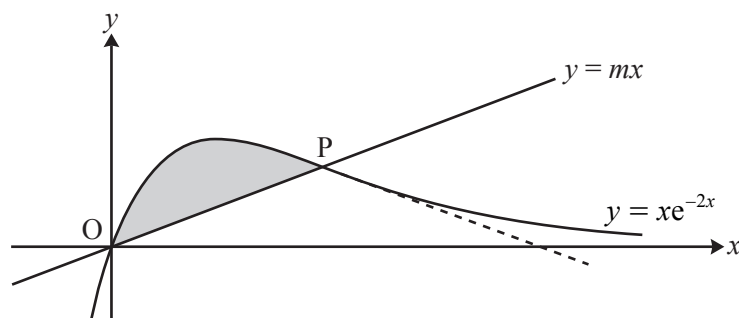


Fig. 9

- (i) Show that the  $x$ -coordinate of P is  $-\frac{1}{2} \ln m$ . [3]

- (ii) Find, in terms of  $m$ , the gradient of the tangent to the curve at P. [4]

You are given that OP and this tangent are equally inclined to the  $x$ -axis.

- (iii) Show that  $m = e^{-2}$ , and find the exact coordinates of P. [4]

- (iv) Find the exact area of the shaded region between the line OP and the curve. [7]

**END OF QUESTION PAPER**



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