Oxford Cambridge and RSA

Friday 12 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Fig. 1 shows part of the curve $y = e^{2x}\cos x$.

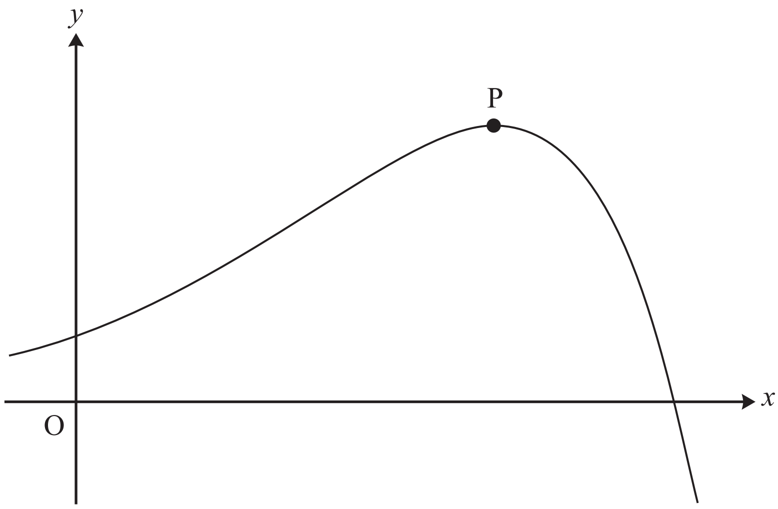

Fig. 1

Find the coordinates of the turning point P. **[6]**

2 Find $\int \sqrt[3]{2x-1}\,dx$. **[4]**

3 Find the exact value of $\int_1^2 x^3 \ln x\,dx$. **[5]**

4 Fig. 4 shows a cone with its axis vertical. The angle between the axis and the slant edge is 45°. Water is poured into the cone at a constant rate of 5 cm^3 per second. At time t seconds, the height of the water surface above the vertex O of the cone is h cm, and the volume of water in the cone is V cm^3.

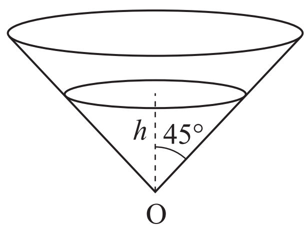

Fig. 4

Find V in terms of h.

Hence find the rate at which the height of water is increasing when the height is 10 cm.

[You are given that the volume V of a cone of height h and radius r is $V = \frac{1}{3}\pi r^2 h$]. **[5]**

5 A curve has implicit equation $y^2 + 2x\ln y = x^2$.

Verify that the point (1, 1) lies on the curve, and find the gradient of the curve at this point. **[6]**

6 Solve each of the following equations, giving your answers in exact form.

(i) $6\arcsin x - \pi = 0$. **[2]**

(ii) $\arcsin x = \arccos x$. **[2]**

7 **(i)** The function $\mathrm{f}(x)$ is defined by

$$\mathrm{f}(x) = \frac{1-x}{1+x},\ x \neq -1.$$

Show that $\mathrm{f}(\mathrm{f}(x)) = x$.

Hence write down $\mathrm{f}^{-1}(x)$. **[3]**

(ii) The function $\mathrm{g}(x)$ is defined for all real x by

$$\mathrm{g}(x) = \frac{1-x^2}{1+x^2}.$$

Prove that $\mathrm{g}(x)$ is even. Interpret this result in terms of the graph of $y = \mathrm{g}(x)$. **[3]**

Section B (36 marks)

8 Fig. 8 shows the line $y = 1$ and the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{(x-2)^2}{x}$. The curve touches the x-axis at P(2, 0) and has another turning point at the point Q.

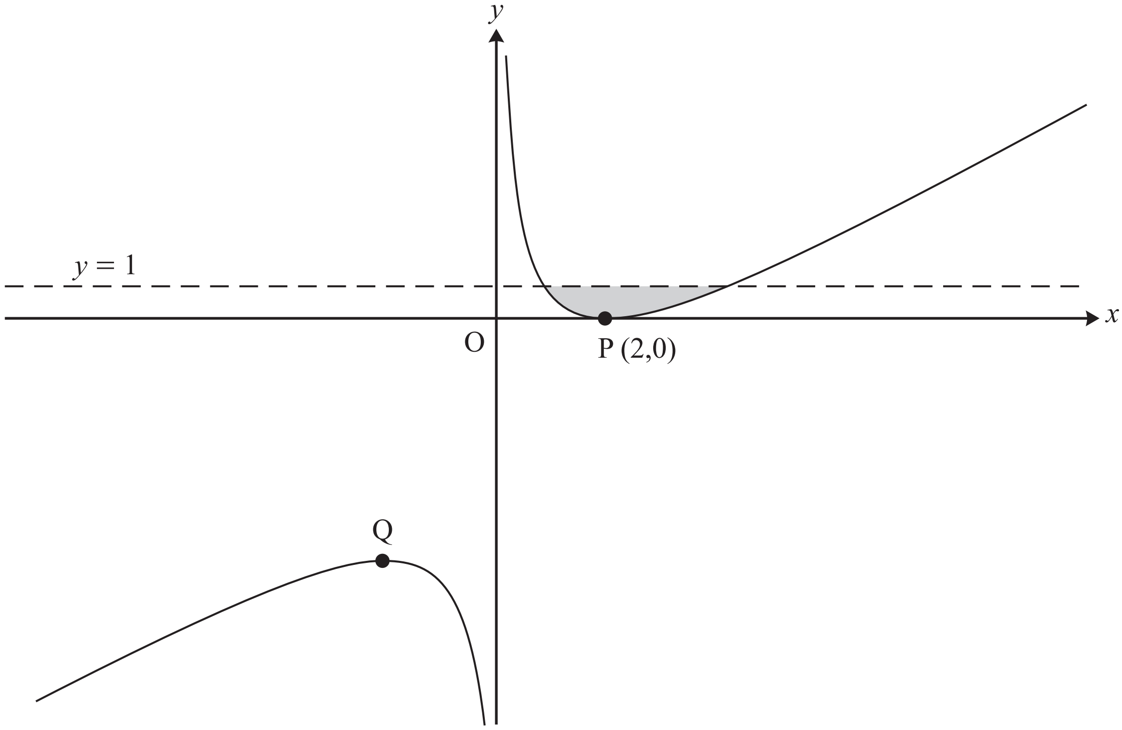

Fig. 8

(i) Show that $\mathrm{f}'(x) = 1 - \dfrac{4}{x^2}$, and find $\mathrm{f}''(x)$.

Hence find the coordinates of Q and, using $\mathrm{f}''(x)$, verify that it is a maximum point. **[7]**

(ii) Verify that the line $y = 1$ meets the curve $y = \mathrm{f}(x)$ at the points with x-coordinates 1 and 4. Hence find the exact area of the shaded region enclosed by the line and the curve. **[6]**

The curve $y = \mathrm{f}(x)$ is now transformed by a translation with vector $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$. The resulting curve has equation $y = \mathrm{g}(x)$.

(iii) Show that $\mathrm{g}(x) = \dfrac{x^2 - 3x}{x+1}$. **[3]**

(iv) Without further calculation, write down the value of $\int_0^3 \mathrm{g}(x)\,\mathrm{d}x$, justifying your answer. **[2]**

9 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (\mathrm{e}^x - 2)^2 - 1, \quad x \in \mathbb{R}.$$

The curve crosses the x-axis at O and P, and has a turning point at Q.

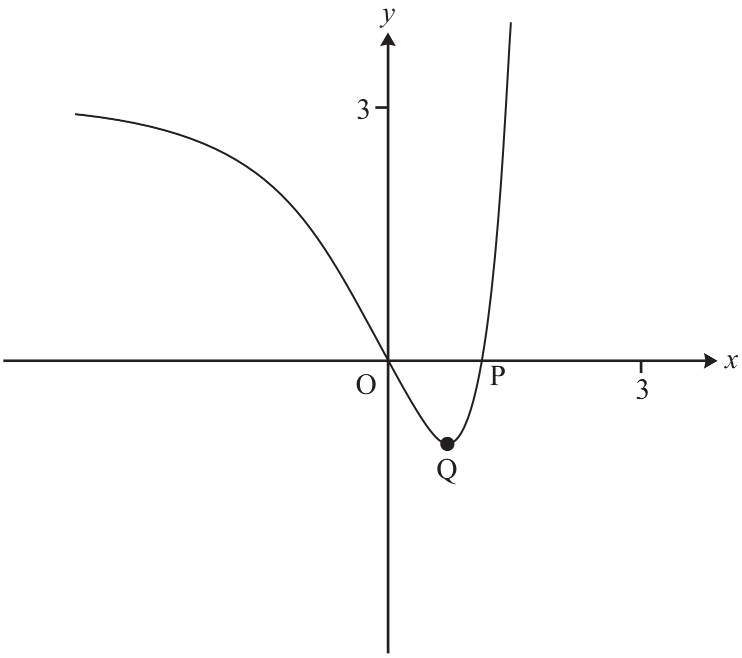

Fig. 9

(i) Find the exact x-coordinate of P. **[2]**

(ii) Show that the x-coordinate of Q is $\ln 2$ and find its y-coordinate. **[4]**

(iii) Find the exact area of the region enclosed by the curve and the x-axis. **[5]**

The domain of $\mathrm{f}(x)$ is now restricted to $x \geqslant \ln 2$.

(iv) Find the inverse function $\mathrm{f}^{-1}(x)$. Write down its domain and range, and sketch its graph on the copy of Fig. 9. **[7]**

END OF QUESTION PAPER

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