

Tuesday 16 June 2015 – Afternoon

A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Section A (36 marks)

- 1 Solve the equation $\frac{5x}{2x+1} - \frac{3}{x+1} = 1$. [5]
- 2 Express $6 \cos 2\theta + \sin \theta$ in terms of $\sin \theta$.
Hence solve the equation $6 \cos 2\theta + \sin \theta = 0$, for $0^\circ \leq \theta \leq 360^\circ$. [7]
- 3 (i) Find the first three terms of the binomial expansion of $\frac{1}{\sqrt[3]{1-2x}}$. State the set of values of x for which the expansion is valid. [5]
(ii) Hence find a and b such that $\frac{1-3x}{\sqrt[3]{1-2x}} = 1 + ax + bx^2 + \dots$. [3]
- 4 You are given that $f(x) = \cos x + \lambda \sin x$ where λ is a positive constant.
(i) Express $f(x)$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving R and α in terms of λ . [4]
(ii) Given that the maximum value (as x varies) of $f(x)$ is 2, find R , λ and α , giving your answers in exact form. [4]
- 5 A curve has parametric equations $x = \sec \theta$, $y = 2 \tan \theta$.
(i) Given that the derivative of $\sec \theta$ is $\sec \theta \tan \theta$, show that $\frac{dy}{dx} = 2 \operatorname{cosec} \theta$. [3]
(ii) Verify that the cartesian equation of the curve is $y^2 = 4x^2 - 4$. [2]

Fig. 5 shows the region enclosed by the curve and the line $x = 2$. This region is rotated through 180° about the x -axis.

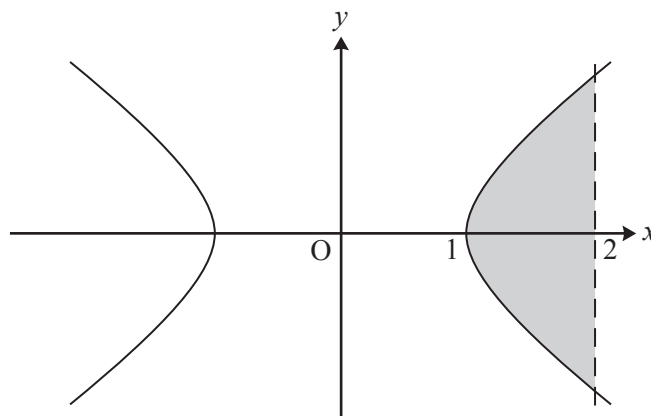


Fig. 5

- (iii) Find the volume of revolution produced, giving your answer in exact form. [3]

Section B (36 marks)

- 6 Fig. 6 shows a lean-to greenhouse ABCDHEFG. With respect to coordinate axes $Oxyz$, the coordinates of the vertices are as shown. All distances are in metres. Ground level is the plane $z = 0$.

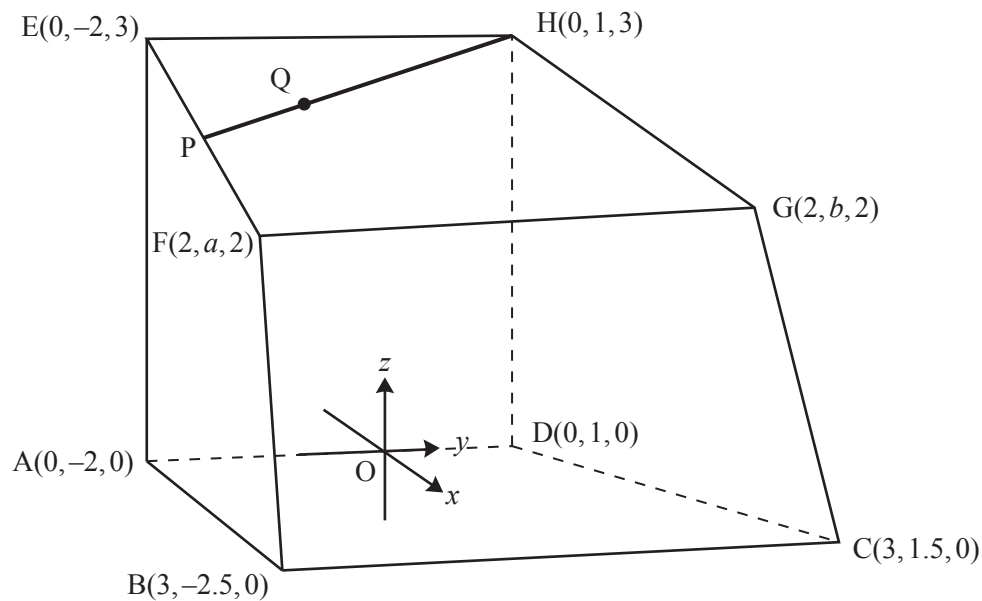


Fig. 6

- (i) Verify that the equation of the plane through A, B and E is $x + 6y + 12 = 0$.

Hence, given that F lies in this plane, show that $a = -2\frac{1}{3}$. [4]

- (ii) (A) Show that the vector $\begin{pmatrix} 1 \\ -6 \\ 0 \end{pmatrix}$ is normal to the plane DHC. [2]

(B) Hence find the cartesian equation of this plane. [2]

(C) Given that G lies in the plane DHC, find b and the length FG. [2]

- (iii) Find the angle EFB. [5]

A straight wire joins point H to a point P which is half way between E and F. Q is a point two-thirds of the way along this wire, so that $HQ = 2QP$.

- (iv) Find the height of Q above the ground. [3]

Question 7 begins on page 4.

- 7 A drug is administered by an intravenous drip. The concentration, x , of the drug in the blood is measured as a fraction of its maximum level. The drug concentration after t hours is modelled by the differential equation

$$\frac{dx}{dt} = k(1 + x - 2x^2),$$

where $0 \leq x < 1$, and k is a positive constant. Initially, $x = 0$.

- (i) Express $\frac{1}{(1+2x)(1-x)}$ in partial fractions. [3]

- (ii) Hence solve the differential equation to show that $\frac{1+2x}{1-x} = e^{3kt}$. [7]

- (iii) After 1 hour the drug concentration reaches 75% of its maximum value and so $x = 0.75$.

Find the value of k , and the time taken for the drug concentration to reach 90% of its maximum value. [3]

- (iv) Rearrange the equation in part (ii) to show that $x = \frac{1 - e^{-3kt}}{1 + 2e^{-3kt}}$.

Verify that in the long term the drug concentration approaches its maximum value. [5]

END OF QUESTION PAPER

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