



Oxford Cambridge and RSA

**Monday 22 June 2015 – Morning**

**A2 GCE MATHEMATICS (MEI)**

**4756/01 Further Methods for Advanced Mathematics (FP2)**

**QUESTION PAPER**



Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration: 1 hour 30 minutes**

**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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## Section A (54 marks)

- 1 (a) (i) A curve has polar equation  $r = 2a \cos \theta + 2b \sin \theta$ , where  $a > 0$  and  $b > 0$ .

Show, by considering its cartesian equation, that the curve is a circle which passes through the origin. Find the centre and radius of the circle in terms of  $a$  and  $b$ . [5]

- (ii) For the case  $a = b = 1$ , use integration to show that the region bounded by a minor arc of the circle and the lines  $\theta = \frac{\pi}{6}$  and  $\theta = \frac{\pi}{3}$  has area  $1 + \frac{\pi}{3}$ . [5]

- (b) Given that  $f(t) = \ln(1+t)$ , obtain expressions for  $f'(t)$ ,  $f''(t)$  and  $f'''(t)$ . Hence show that the Maclaurin series for  $\ln(1+t)$  begins

$$t - \frac{t^2}{2} + \frac{t^3}{3} \dots$$

Deduce the first two non-zero terms of the Maclaurin series for  $\ln\left(\frac{1+t}{1-t}\right)$ . [8]

- 2 (a) (i) By considering  $\left(z + \frac{1}{z}\right)^5$ , where  $z = \cos \theta + j \sin \theta$ , show that

$$\cos^5 \theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos \theta). \quad [5]$$

- (ii) Use de Moivre's theorem to find an expression for  $\cos 5\theta$  in terms of powers of  $\cos \theta$ . [5]

- (b) (i) Obtain the roots of the equation  $w^5 = 4\sqrt{2}$  in the form  $re^{j\theta}$ . Show the points corresponding to these roots in an Argand diagram. [4]

- (ii) For each root  $w$ , let  $v = w\sqrt{2}e^{j\pi/10}$ .

Show the points corresponding to the values of  $v$  on your Argand diagram.

Find, in simplified form, an equation for which the values of  $v$  are the roots. [4]

- 3 This question concerns the matrix  $\mathbf{M}$  where  $\mathbf{M} = \begin{pmatrix} 5 & -1 & 3 \\ 4 & -3 & -2 \\ 2 & 1 & 4 \end{pmatrix}$ .

- (i) Obtain the characteristic equation of  $\mathbf{M}$ .

Find the eigenvalues of  $\mathbf{M}$ .

[7]

These eigenvalues are denoted by  $\lambda_1, \lambda_2, \lambda_3$ , where  $\lambda_1 < \lambda_2 < \lambda_3$ .

- (ii) Verify that an eigenvector corresponding to  $\lambda_1$  is  $\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$  and that an eigenvector corresponding to  $\lambda_2$  is

$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ . Find an eigenvector of the form  $\begin{pmatrix} a \\ 1 \\ c \end{pmatrix}$  corresponding to  $\lambda_3$ .

[5]

- (iii) Write down a matrix  $\mathbf{P}$  and a diagonal matrix  $\mathbf{D}$  such that  $\mathbf{M} = \mathbf{PDP}^{-1}$ . (You are not required to calculate  $\mathbf{P}^{-1}$ .)

Hence write down an expression for  $\mathbf{M}^4$  in terms of  $\mathbf{P}$  and a diagonal matrix. You should give the elements of the diagonal matrix explicitly.

[3]

- (iv) Use the Cayley-Hamilton theorem to obtain an expression for  $\mathbf{M}^4$  as a linear combination of  $\mathbf{M}$  and  $\mathbf{M}^2$ .

[3]

### Section B (18 marks)

- 4 (i) Starting with the relationship  $\cosh^2 t - \sinh^2 t = 1$ , deduce a relationship between  $\tanh^2 t$  and  $\operatorname{sech}^2 t$ .

You are given that  $y = \operatorname{artanh} x$ .

- (ii) Show that  $\frac{dy}{dx} = \frac{1}{1-x^2}$ .

[4]

- (iii) Show, by integrating the result in part (ii), that  $y = \frac{1}{2} \ln \left( \frac{1+x}{1-x} \right)$ .

[4]

- (iv) Show that  $\int_0^{\frac{\sqrt{3}}{6}} \frac{1}{1-3x^2} dx = \frac{1}{\sqrt{3}} \operatorname{artanh} \frac{1}{2}$ . Express this answer in logarithmic form.

[4]

- (v) Use integration by parts to find  $\int \operatorname{artanh} x dx$ , giving your answer in terms of logarithms.

[5]

**END OF QUESTION PAPER**



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