OCR
Oxford Cambridge and RSA

Wednesday 3 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4757/01 Further Applications of Advanced Mathematics (FP3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4757/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **24** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Option 1: Vectors

1 The point A has coordinates (2, 5, 4) and the line BC has equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 25 \\ 43 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 15 \\ 25 \end{pmatrix}.$$

You are given that AB = AC = 15.

(i) Show that the coordinates of one of the points B and C are (4, 10, 18). Find the coordinates of the other point. These points are B and C respectively. **[6]**

(ii) Find the equation of the plane ABC in cartesian form. **[4]**

(iii) Show that the plane containing the line BC and perpendicular to the plane ABC has equation $5y - 3z + 4 = 0$. **[4]**

The point D has coordinates (1, 1, 3).

(iv) Show that $|\overrightarrow{BC} \times \overrightarrow{AD}| = \sqrt{7667}$ and hence find the shortest distance between the lines BC and AD. **[7]**

(v) Find the volume of the tetrahedron ABCD. **[3]**

Option 2: Multi-variable calculus

2 A surface has equation $z = 3x^2 - 12xy + 2y^3 + 60$.

(i) Show that the point A $(8, 4, -4)$ is a stationary point on the surface. Find the coordinates of the other stationary point, B, on this surface. **[5]**

(ii) A point P with coordinates $(8+h, 4+k, p)$ lies on the surface.

(A) Show that $p = -4 + 3(h - 2k)^2 + 2k^2(6+k)$. **[3]**

(B) Deduce that the stationary point A is a local minimum. **[3]**

(C) By considering sections of the surface near to B in each of the planes $x = 0$ and $y = 0$, investigate the nature of the stationary point B. **[4]**

(iii) The point Q with coordinates (1, 1, 53) lies on the surface.

Show that the equation of the tangent plane at Q is

$$6x + 6y + z = 65.$$ **[4]**

(iv) The tangent plane at the point R has equation $6x + 6y + z = \lambda$ where $\lambda \neq 65$.

Find the coordinates of R. **[5]**

Option 3: Differential geometry

3 Fig. 3 shows an ellipse with parametric equations $x = a\cos\theta$, $y = b\sin\theta$, for $0 \leqslant \theta \leqslant 2\pi$, where $0 < b \leqslant a$.
The curve meets the positive x-axis at A and the positive y-axis at B.

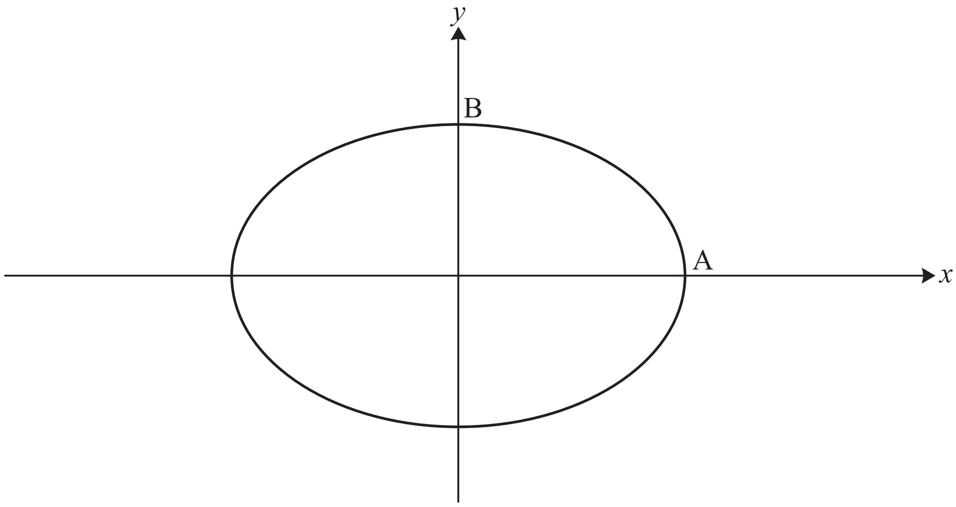

Fig. 3

(i) Show that the radius of curvature at A is $\dfrac{b^2}{a}$ and find the corresponding centre of curvature. **[7]**

(ii) Write down the radius of curvature and the centre of curvature at B. **[2]**

(iii) Find the relationship between a and b if the radius of curvature at B is equal to the radius of curvature at A. What does this mean geometrically? **[1]**

(iv) Show that the arc length from A to B can be expressed as

$$b\int_0^{\frac{\pi}{2}} \sqrt{1+\lambda^2\sin^2\theta}\, d\theta,$$

where λ^2 is to be determined in terms of a and b.

Evaluate this integral in the case $a = b$ and comment on your answer. **[7]**

(v) Find the cartesian equation of the evolute of the ellipse. **[7]**

Option 4: Groups

4 M is the set of all 2×2 matrices m(a,b) where a and b are rational numbers and

$$m(a,b) = \begin{pmatrix} a & b \\ 0 & \frac{1}{a} \end{pmatrix}, a \neq 0.$$

(i) Show that under matrix multiplication M is a group. You may assume associativity of matrix multiplication. **[7]**

(ii) Determine whether the group is commutative. **[3]**

The set N_k consists of all 2×2 matrices m(k,b) where k is a fixed positive integer and b can take any integer value.

(iii) Prove that N_k is closed under matrix multiplication if and only if $k = 1$. **[4]**

Now consider the set P consisting of the matrices m(1,0), m(1,1), m(1,2) and m(1,3). The elements of P are combined using matrix multiplication but with arithmetic carried out modulo 4.

(iv) Show that $(m(1,1))^2 = m(1,2)$. **[2]**

(v) Construct the group combination table for P. **[4]**

The group R consists of the set $\{e, a, b, c\}$ combined under the operation *. The identity element is e, and elements a, b and c are such that

$$a*a = b*b = c*c \qquad \text{and} \qquad a*c = c*a = b.$$

(vi) Determine whether R is isomorphic to P. **[4]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 An inspector has three factories, A, B, C, to check. He spends each day in one of the factories. He chooses the factory to visit on a particular day according to the following rules.

- If he is in A one day, then the next day he will never choose A but he is equally likely to choose B or C.
- If he is in B one day, then the next day he is equally likely to choose A, B or C.
- If he is in C one day, then the next day he will never choose A but he is equally likely to choose B or C.

(i) Write down the transition matrix, **P**. **[2]**

(ii) On Day 1 the inspector chooses A.

(A) Find the probability that he will choose A on Day 4. **[3]**

(B) Find the probability that the factory he chooses on Day 7 is the same factory that he chose on Day 2. **[4]**

(iii) Find the equilibrium probabilities and explain what they mean. **[4]**

The inspector is not satisfied with the number of times he visits A so he changes the rules as follows.

- If he is in A one day, then the next day he will choose A, B, C, with probabilities 0.8, 0.1, 0.1, respectively.
- If he is in B or C one day, then the probabilities for choosing the factory the next day remain as before.

(iv) Write down the new transition matrix, **Q**, and find the new equilibrium probabilities. **[3]**

(v) On a particular day, the inspector visits factory A. Find the expected number of consecutive further days on which he will visit factory A. **[3]**

Still not satisfied, the inspector changes the rules as follows.

- If he is in A one day, then the next day he will choose A, B, C, with probabilities 1, 0, 0, respectively.
- If he is in B or C one day, then the probabilities for choosing the factory the next day remain as before.

The new transition matrix is **R**.

(vi) On Day 15 he visits C. Find the first subsequent day for which the probability that he visits B is less than 0.1. **[3]**

(vii) Show that in this situation there is an absorbing state, explaining what this means. **[2]**

END OF QUESTION PAPER

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