OCR
Oxford Cambridge and RSA

Friday 12 June 2015 – Morning

AS GCE MATHEMATICS (MEI)

4776/01 Numerical Methods

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
*   Printed Answer Book 4776/01
*   MEI Examination Formulae and Tables (MF2)

Other materials required:
*   Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

*   The Question Paper will be found inside the Printed Answer Book.
*   Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
*   **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
*   Use black ink. HB pencil may be used for graphs and diagrams only.
*   Read each question carefully. Make sure you know what you have to do before starting your answer.
*   Answer **all** the questions.
*   Do **not** write in the bar codes.
*   You are permitted to use a scientific or graphical calculator in this paper.
*   Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

*   The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
*   You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
*   The total number of marks for this paper is **72**.
*   The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Turn over

Section A (36 marks)

1   **(i)** Show that the equation

$$x = (\cos x)^3 \qquad (*)$$

where x is in radians, has a root in the interval $(0, 1)$. **[2]**

This root is denoted by α.

  (ii) Show numerically that the iteration

$$x_{r+1} = (\cos x_r)^3$$

with $x_0 = 0.6$ does not converge to α. **[2]**

(iii) Show that

$$x = \sqrt{x(\cos x)^3}$$

is a rearrangement of (*). Use the corresponding iteration to find α correct to 4 decimal places. **[4]**

2   An estimate is required of I, where

$$I = \int_0^{0.6} f(x)\,dx.$$

The only available values of $f(x)$ are as follows.

x	0	0.1	0.2	0.4
$f(x)$	2.3063	2.2769	2.1883	1.8308

  (i) Obtain the best possible estimates of $\int_0^{0.2} f(x)\,dx$ and $\int_{0.2}^{0.6} f(x)\,dx$. Hence give an estimate of I. **[5]**

  (ii) State what you would do differently if $f(0.6)$ became available. **[1]**

3

3 A computer program is used to calculate values of *f* where

$$\frac{1}{f} = \frac{1}{u} - \frac{1}{v}.$$

One possible formula for *f* is

$$f = \frac{1}{\left(\frac{1}{u} - \frac{1}{v}\right)}. \qquad (*)$$

(i) Show that another formula for *f* is

$$f = \frac{uv}{v - u}. \qquad (**)$$ **[1]**

The program stores and calculates all numbers rounded to 5 significant figures.

(ii) Find the values of *f* given by the program using (*) and (**) when *u* = 11 and *v* = 11.05.

Show that one of these values is exact and find the relative error in the other. **[6]**

(iii) State what process gives rise to the error in the inexact value. **[1]**

4 The table shows values of a function f(*x*) correct to 5 decimal places.

x	1	1.1	1.01	1.001	1.0001	1.00001
f(*x*)	0.944 04	0.967 71	0.946 41	0.944 28	0.944 06	0.944 04

(i) Find five estimates of f′(1) using the forward difference method. **[3]**

(ii) Comment on the accuracy of these estimates.

Give a value for f′(1) to the accuracy that you consider appropriate. Justify your answer. **[3]**

5 An approximate formula for $\sqrt{x}$ of the form

$$\sqrt{x} \approx a + bx \qquad (*)$$

is required for values of *x* near to 1.

(i) Find the values of *a* and *b* for which (*) is exact when *x* = 1 and *x* = 1.21. **[4]**

(ii) With these values of *a* and *b*, find the absolute and relative errors in (*) when *x* = 0.81. **[4]**

Section B (36 marks)

6 The variables p and q are known to take the following values.

p	1	2	3
q	2.2	2.8	5.2

    (i) Plot these points and draw, by eye, a smooth curve through them. **[2]**

    (ii) Use Newton's forward difference interpolation formula to obtain a quadratic expression for q in terms of p. Give your answer in simplified form. **[6]**

    (iii) Estimate q when $p = 2.5$. Let this estimate be denoted by α. **[2]**

    (iv) Now suppose that a quadratic expression for p in terms of q is required. Explain why Newton's formula could not be used for this purpose. Use Lagrange's method to write down an expression for this quadratic. (You are not required to simplify this expression.) **[5]**

    (v) Estimate p when q is equal to the value α found in part **(iii)**. Comment on your answer. **[3]**

7   **(i)** Show that the equation

$$3x^4 + x - 2 = 0 \qquad (*)$$

has a root, α, in the interval $(0, 1)$.

Show that (*) does not have any other positive roots. **[3]**

    (ii) Sketch the curve $y = 3x^4 + x - 2$ for $0 \leqslant x \leqslant 1$. **[2]**

The secant method, with $x_0 = 0$ and $x_1 = 1$ is used to find α.

    (iii) Find x_2 exactly, and calculate x_3 and x_4 correct to 6 significant figures.

Show, by means of appropriate lines on your graph, how the secant method produces these values. **[8]**

    (iv) Iterate the secant method further to find α correct to 3 significant figures. Show that you have obtained the required accuracy. **[5]**

END OF QUESTION PAPER

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