GCE

Mathematics (MEI)

Unit **4754A**: Applications of Advanced Mathematics: Paper A

Advanced GCE

Mark Scheme for June 2015

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

1. **Annotations and abbreviations**

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. **Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand**

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1			$\Rightarrow 5x(x + 1) – 3(2x + 1) = (2x + 1)(x + 1)$	M1*	Multiplying throughout by $(2x + 1)(x + 1)$ or combining fractions and multiplying up oe (eg can retain denominator throughout) Condone a single numerical error, sign error or slip provided that there is no conceptual error in the process involved Do not condone omission of brackets unless it is clear from subsequent work that they were assumed eg $5x(x + 1) – 3(2x + 1) = (2x + 1)(x \underline{– 1})$ gets M1 $5x(x + 1) – 3(2x + 1) = 1$ gets M0 $5x(x + 1)(2x + 1) – 3(2x + 1)(x + 1) = (x + 1)(2x + 1)$ gets M0 $5x(x + 1) – 3(2x + 1) = (2x + 1)$ gets M1, just, for slip in omission of $(x + 1)$
				M1dep*	Multiplying out, collecting like terms and forming quadratic (= 0). Follow through from their equation provided the algebra is not significantly eased and it is a quadratic. Condone a further sign or numerical error or a minor slip when rearranging
			$\Rightarrow \quad 3x^2 – 4x – 4 = 0$	A1	oe www (not fortuitously obtained – check for double errors)
			$\Rightarrow \quad (3x + 2)(x – 2) = 0$	M1	Solving their three term quadratic (= 0) provided $b^2 - 4ac \geq 0$. Use of correct quadratic equation formula (if formula is quoted correctly then only one sign slip is permitted, if the formula is quoted incorrectly M0, if not quoted at all substitution must be completely correct to earn the M1) or factorising (giving their x^2 term and one other term when factors multiplied out) or comp. the square (must get to the square root stage involving $\pm$ and arithmetical errors may be condoned provided their $3\left(x-2/3\right)^2$ seen or implied)
			$\Rightarrow \quad x = -2/3$ or 2	A1	cao for both obtained www (condone – 0.667 or better) (If no factorisation (oe) seen B1 for each answer stated following correct quadratic)
				[5]	

Question			Answer	Marks	Guidance
2			$\cos 2\theta = 1 - 2\sin^2\theta$	M1*	$\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ (maybe implied in substitution)
			$(6\cos 2\theta + \sin\theta =)\ 6 - 12\sin^2\theta + \sin\theta$	A1	
			$6\cos 2\theta + \sin\theta = 0$ $\Rightarrow \quad 12\sin^2\theta - \sin\theta - 6 = 0$ $\Rightarrow \quad (4\sin\theta - 3)(3\sin\theta + 2) = 0$	M1dep*	Use of correct quadratic equation formula or factorising or comp. the square on their three term quadratic in $\sin\theta$ (see guidance in question 1 for awarding this method mark) provided $b^2 - 4ac \geq 0$
			$\Rightarrow \quad \sin\theta = 3/4$ or $-2/3$	A1	www
				B1	First correct solution to 1 dp or better (eg 48.59° etc)
				B1	Three correct solutions
			$\Rightarrow \quad \sin\theta = 3/4$, $\theta = 48.6°$, $131.4°$ $\sin\theta = -2/3$, $\theta = 221.8°$, $318.2°$	B1	All four correct solutions and no others in the range Ignore solutions outside the range **SC** Award max B1B1B0 for answers in radians (0.85, 2.29, 3.87, 5.55 or better – so one correct B1, three correct B1). Award max B1 if there are extra solutions in the range with radians **SC** If M1M1 awarded and both values of $\lvert\sin\theta\rvert \leq 1$ but B0B0B0 then award B1 only for evidence of using $\sin\theta \equiv \sin(180 - \theta)$
				[7]	

Question			Answer	Marks	Guidance
3	(i)		$\frac{1}{\sqrt[3]{1-2x}}=(1-2x)^{-1/3}$	B1	n = –1/3. See below **SC** for those with n = 1/3
			$=1+(-\frac{1}{3})(-2x)+\frac{(-\frac{1}{3})(-\frac{4}{3})}{2!}(-2x)^2+...$	M1	All three **correct unsimplified binomial** coefficients (not nCr) soi condone absence of brackets only if it is clear from subsequent work that they were assumed
				B1	$1+(2/3)x+...$ www
			$=1+\frac{2}{3}x+\frac{8}{9}x^2+...$	B1	$(8/9)x^2$ www in this term If there is an error, in say, the third coefficient of the expansion then M0B1B0 is possible **SC** For n = 1/3 award B1 for $1-(2/3)x$ and B1 for $-(4/9)x^2$ (so max 2 out of the first 4 marks)
			Valid for $-\frac{1}{2}<x<\frac{1}{2}$ or $\lvert x\rvert<\frac{1}{2}$	B1 **[5]**	Independent of expansion. Accept, say, $-1/2<\lvert x\rvert<1/2$ or $-1/2\le x<1/2$ (must be strict inequality for +1/2)
3	(ii)		$\frac{1-3x}{\sqrt[3]{1-2x}}=(1-3x)(1+\frac{2}{3}x+\frac{8}{9}x^2+...)$		
			$=1+\frac{2}{3}x+\frac{8}{9}x^2-3x-2x^2+...$	M1	Use of $(1-3x)\times$ their$\left(1+(2/3)x+(8/9)x^2+\cdots\right)$ and attempt at removal of brackets (condone absence of brackets but must have two terms in x and two terms in x^2)
				A1ft	Correct simplified expansion following their expansion in (i). This mark is dependent on scoring both M marks in (i) and (ii)
			$=1-\frac{7}{3}x-\frac{10}{9}x^2+...$	A1 **[3]**	cao or B3 www in either part **SC** following either M0 or M1, B1 for either a or b correct

Question			Answer	Marks	Guidance
4	(i)		$\cos x + \lambda \sin x = R\cos(x - \alpha)$ $= R\cos x\cos\alpha + R\sin x\sin\alpha$		
			$\Rightarrow$ $R\cos\alpha = 1$, $R\sin\alpha = \lambda$	M1	Correct pairs. Condone sign error (so accept $R\sin\alpha = -\lambda$)
			$\Rightarrow$ $R^2 = 1 + \lambda^2$, $R = \sqrt{(1 + \lambda^2)}$	B1	Positive square root only - isw. Accept $R = 1/\cos(\arctan\lambda)$ or $R = \lambda/\sin(\arctan\lambda)$
			$\tan\alpha = \lambda$ (oe)	M1	Follow through their pairs. $\tan\alpha = \lambda$ with no working implies both M marks. However, $\cos\alpha = 1$, $\sin\alpha = \lambda \Rightarrow \tan\alpha = \lambda$ scores M0M1. First two M marks may be implied by combining one of the pairs with R, eg, $\cos\alpha = \frac{1}{\sqrt{(1+\lambda^2)}}$ or $\sin\alpha = \frac{\lambda}{\sqrt{(1+\lambda^2)}}$
			$\Rightarrow \alpha = \arctan\lambda$ (oe)	A1	$\alpha = \arccos\left(\frac{1}{\sqrt{(1+\lambda^2)}}\right)$, $\alpha = \arcsin\left(\frac{\lambda}{\sqrt{(1+\lambda^2)}}\right)$ Accept embedded answers, eg, $\sqrt{(1+\lambda^2)}\cos(x - \arctan\lambda)$ for full marks
				[4]	
4	(ii)		max is R so $R = 2$	B1	
			$1 + \lambda^2 = 4 \Rightarrow \lambda = \sqrt{3}$	M1 A1	M1 for using their $\sqrt{(1+\lambda^2)} = R_{max}$, A0 for $\pm\sqrt{3}$ as final answer
			$\alpha = \arctan\sqrt{3} = \pi/3$	B1	www (eg $\lambda = 1$ and $\cos\alpha = (1+\lambda)^{-1} \Rightarrow \alpha = \pi/3$ is B0) Exact answers only for final A and B marks
				[4]	

Question			Answer	Marks	Guidance
5	(i)		$x = \sec\theta,\ y = 2\tan\theta$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{2\sec^2\theta}{\sec\theta\tan\theta}$ $= \dfrac{2\sec\theta}{\tan\theta} = \dfrac{2}{\cos\theta}.\dfrac{\cos\theta}{\sin\theta} = \dfrac{2}{\sin\theta} = 2\operatorname{cosec}\theta$ *	M1A1 A1 **[3]**	M1 for their $(dy/d\theta)\div\sec\theta\tan\theta$ in terms of θ A1 cao (oe) allow for unsimplified form even if subsequently cancelled incorrectly ie can isw cao www (NB **AG**) – must be at least one intermediate step between $\dfrac{2\sec\theta}{\tan\theta}$ (oe) and either $\dfrac{2}{\sin\theta}$ or $2\operatorname{cosec}\theta$
5	(ii)		$x^2 = \sec^2\theta = 1 + \tan^2\theta = 1 + \frac{1}{4}y^2$ $\Rightarrow\ y^2 = 4(x^2 - 1) = 4x^2 - 4$ *	M1 A1 **[2]**	$\sec^2\theta = 1 + \tan^2\theta$ (oe) used www NB **AG**
		OR	$4\tan^2\theta = 4\sec^2\theta - 4$ $\Rightarrow 1 + \tan^2\theta = \sec^2\theta$ which is true	B1* B1dep*	Correct substitution of x and y into the given answer Dependent on previous mark – must simplify/remove the factor of 4 from each term and state that the correctly derived trig identity is true
5	(iii)		$V = \pi\int_1^2 y^2\,dx = \pi\int_1^2 (4x^2-4)\,dx$ $\frac{4}{3}x^3 - 4x$ $= \pi\left[\frac{4}{3}x^3 - 4x\right]_1^2 = \frac{16}{3}\pi$	M1 B1 A1 **[3]**	$k\pi\int_1^2 (4x^2-4)(dx)$ with $k = 1$ or 1/2, allow correct limits later condone lack of dx $(4/3)x^3 - 4x$ $\left(\text{or } (2/3)x^3 - 2x\right)$ exact – mark **final** answer

Question			Answer	Marks	Guidance
6	(i)		A: 0 + 6.(−2) + 12 = 0 B: 3 + 6.(−2.5) + 12 = 0 E: 0 + 6.(−2) + 12 = 0	B2,1,0	B1 for two points verified (must see as a minimum –12 + 12 = 0, 3 – 15 + 12 = 0, –12 + 12 = 0) or any valid complete method for either finding or verifying that $x+6y+12=0$ gets M1 A1
			At F, 2 + 6*a* + 12 = 0	M1	Substitution of F into $x+6y+12=0$
			⇒ 6*a* = −14, *a* = −14/6 = −7/3 *	A1	www NB **AG**
				[4]	
6	(ii)	(A)	$\overline{\text{DH}}.\begin{pmatrix}1\\-6\\0\end{pmatrix}=\begin{pmatrix}1\\-6\\0\end{pmatrix}\begin{pmatrix}0\\0\\3\end{pmatrix}=1\times0+(-6)\times0+0\times3=0$	B1	scalar product with a **direction** vector in the plane (including evaluation and = 0) (**OR** M1 forms a vector product with at least two correct terms in solution)
			$\overline{\text{DC}}.\begin{pmatrix}1\\-6\\0\end{pmatrix}=\begin{pmatrix}3\\0.5\\0\end{pmatrix}.\begin{pmatrix}1\\-6\\0\end{pmatrix}=3\times1+0.5\times(-6)+0\times0=0$	B1	scalar product with second **direction** vector, with evaluation. (following OR above, A1 correct ie a multiple of $\mathbf{i}-6\mathbf{j}$) (NB finding only one direction vector and its scalar product is B1 only)
				[2]	
6	(ii)	(B)	**r**.(**i** – 6**j**) = **j**.(**i** – 6**j**)	M1	$\mathbf{r}\cdot\mathbf{n}=\mathbf{a}\cdot\mathbf{n}$ with $\mathbf{n}=\begin{pmatrix}1\\-6\\0\end{pmatrix}$ and $\mathbf{a}=\begin{pmatrix}0\\1\\3\end{pmatrix}$ or $\begin{pmatrix}0\\1\\0\end{pmatrix}$ or $\begin{pmatrix}3\\1.5\\0\end{pmatrix}$ **or** substituting H(0, 1, 3) or D(0, 1, 0) or C(3, 1.5, 0) into $x-6y=d$
			⇒ *x* – 6*y* + 6 = 0	A1	oe (isw if *d* found correctly and *x* – 6y = *d* stated) B2 www correct equation stated
				[2]	
6	(ii)	(C)	2 −6b + 6 = 0 ⇒ b = 4/3	B1	oe – exact answer
			$\text{FG}=1\frac{1}{3}+2\frac{1}{3}=3\frac{2}{3}$	B1	oe – exact answer
				[2]	

Question			Answer	Marks	Guidance
6	(iii)		($\overrightarrow{FE}$ =) −2**i** + (1/3)**j** + **k**, ($\overrightarrow{FB}$ =) **i** – (1/6)**j** – 2**k**	B1 B1	or $(\overrightarrow{EF}=)2\mathbf{i}+(-1/3)\mathbf{j}-\mathbf{k}$ or $(\overrightarrow{BF}=)-\mathbf{i}+(1/6)\mathbf{j}+2\mathbf{k}$
			$\cos\theta=\dfrac{-2(1)+(1/3)(-1/6)+1(-2)}{\sqrt{4+1/9+1}\sqrt{1+1/36+4}}$	M1	$\cos\theta=\left(\overrightarrow{FE}\cdot\overrightarrow{FB}\right)/\left(\left\|\overrightarrow{FE}\right\|\left\|\overrightarrow{FB}\right\|\right)$ (oe) follow through their FE and FB (allow any combination of FE, EF with FB, BF) – allow one sign slip only
			$\theta=\arccos\left(\dfrac{-\frac{73}{18}}{\frac{\sqrt{46}}{3}\times\frac{\sqrt{181}}{6}}\right)$	A1	$\arccos\left(\pm\dfrac{-2-1/18-2}{5.069}\right)=\arccos\left(\pm-0.800\right)$
			⇒ q = 143°	A1 **[5]**	3sf or better (or 2.5(0) radians or better). Allow candidates who find the acute angle using either $\overrightarrow{EF}$ with $\overrightarrow{FB}$ or $\overrightarrow{FE}$ with $\overrightarrow{BF}$ and then state the obtuse angle. **Do not isw those who find the obtuse angle and then state the acute angle.** Note: $90+2\arctan(1/2)$ is 0/5
		OR	EF = $\sqrt{46}/3$, FB = $\sqrt{181}/6$, EB = $\sqrt{73}/2$	B3,2,1,0	One mark for each (2.26, 2.24, 4.27)
			$\theta=\arccos\left(\dfrac{\left(\sqrt{46}/3\right)^2+\left(\sqrt{181}/6\right)^2-\left(\sqrt{73}/2\right)^2}{2(\sqrt{46}/3)(\sqrt{181}/6)}\right)$	M1 A1	cosine rule correct with their EF, FB, EB q = 143°
6	(iv)		*z* coordinate of P is 5/2	B1	stating the correct *z*-coordinate of P; ignore incorrect *x* and *y* coordinates (or stated in a position vector)
			$\overrightarrow{OQ}=\overrightarrow{OP}+\overrightarrow{PQ}=\begin{pmatrix}1\\-13/6\\5/2\end{pmatrix}+\dfrac{1}{3}\left(\begin{pmatrix}0\\1\\3\end{pmatrix}-\begin{pmatrix}1\\-13/6\\5/2\end{pmatrix}\right)$	M1	Complete method for finding the z-coordinate of Q or $\overrightarrow{OQ}=(\overrightarrow{OH})+(2/3)(\overrightarrow{HP})$ or $\overrightarrow{OQ}=(2/3)(\overrightarrow{OP})+(1/3)(\overrightarrow{OH})$
			so height of Q is 8/3 (metres above ground)	A1 **[3]**	2.67 or better

Question			Answer	Marks	Guidance
7	(i)		$\frac{1}{(1+2x)(1-x)}=\frac{A}{1+2x}+\frac{B}{1-x}\Rightarrow 1=A(1-x)+B(1+2x)$		
				M1	Cover up, substitution or equating coefficients
			$x = 1 \Rightarrow 3B = 1$, $B = 1/3$	A1	
			$x = -½ \Rightarrow 1 = 3A/2$, $A = 2/3$	A1	isw after correct A and B stated
				[3]	
7	(ii)		$1+x-2x^2=(1+2x)(1-x)$	B1	May be seen in separation of variables (may be implied by later working) – implied by the use of factors (1 + 2*x*) and (1 – *x*)
			$\Rightarrow \quad \frac{1}{3}\int[\frac{2}{(1+2x)}+\frac{1}{1-x}]\mathrm{d}x=\int k\mathrm{d}t$	M1	Separating variables and substituting partial fractions. If no subsequent work integral signs needed, but allow omission of $\mathrm{d}x$ or $\mathrm{d}t$, but must be correctly placed if present
			$\lambda\ln(1+2x)+\mu\ln(1-x)=kt(+c)$	A1	Any non-zero constant λ, μ
			$\Rightarrow \quad \ln(1 + 2x) - \ln(1 - x) = 3kt\ (+ c)$	A1	www oe (condone absence of *c*)
			When $t = 0$, $x = 0 \Rightarrow c = 0$	B1	cao (**must** follow previous A1) need to **show** (at some stage) that *c* = 0. As a minimum *t* = 0, *x* = 0, *c* = 0. Note that *c* = ln(-1) (usually from incorrect integration of (1 – *x*)) or similar scores B0
			$\Rightarrow \quad \ln\left(\frac{1+2x}{1-x}\right)=3kt$	M1	Combining both their log terms correctly. Follow through their c. Allow if *c* = 0 clearly stated (provided that *c* = 0) even if B mark is not awarded, but do not allow if *c* omitted
			$\Rightarrow \quad \frac{1+2x}{1-x}=\mathrm{e}^{3kt}$ *	A1	**AG** www must have obtained all previous marks in this part
				[7]	

Question			Answer	Marks	Guidance
7	(iii)		$(1 + 2(0.75)) / (1 – 0.75) = e^{3k}$	M1	substituting $t = 1$, $x = 0.75$ at any stage
			$k = (1/3)\ln 10$ (= 0.768 (3 s.f.))	A1	3sf or better
			$t = \ln(2.8/0.1)/3k = 1.45$ hours	A1	1.45 (or better) or 1 hr 27 mins
				[3]	
7	(iv)		$1 + 2x = e^{3kt} – xe^{3kt}$		
			$\Rightarrow \quad 2x + xe^{3kt} = e^{3kt} – 1$	M1*	Multiplying out and collecting x terms (condone one error)
			$\Rightarrow \quad x(2 + e^{3kt}) = e^{3kt} – 1$	M1dep*	Factorising their x terms correctly
			$\Rightarrow \quad x = (e^{3kt} – 1)/ (2 + e^{3kt})$	A1	
			$= (1 – e^{-3kt})/(1 + 2e^{-3kt})$ *	A1	www (**AG**) – as AG must be an indication of how previous line leads to the required result (eg stating or showing multiplying by e^{-3kt})
			when $t \to \infty \quad e^{-3kt} \to 0$ $x = (1 – e^{-3kt})/(1 + 2e^{-3kt}) \to 1/1 = 1$	B1	clear indication that $e^{-3kt} \to 0$ so, for example, accept as a minimum $(x \to)\dfrac{1-0}{1+0} = 1$ **or** $e^{-3kt} \to 0 \Rightarrow (x \to)\, 1$ (NB substitution of large values of t with no further explanation is B0)
				[5]	
		OR	$\dfrac{1-x}{1+2x} = e^{-3kt}$	B1	
			$1 - x = e^{-3kt} + 2xe^{-3kt}$	M1*	Multiplying up and expanding (condone one error)
			$x(1 + 2e^{-3kt}) = 1 - e^{-3kt}$	M1dep*	Factorising their x terms correctly
			$x = (1 – e^{-3kt})/(1 + 2e^{-3kt})$ *	A1	www (**AG**) – final B mark as in scheme above

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