OCR
Oxford Cambridge and RSA

Tuesday 21 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Find the exact value of $\int_0^{\frac{1}{2}\pi} (1+\cos\frac{1}{2}x)\,\mathrm{d}x$. **[3]**

2 The functions f(x) and g(x) are defined by f(x) = ln x and g(x) = 2 + e^x, for $x > 0$.

Find the exact value of x, given that fg(x) = 2x. **[5]**

3 Find $\int_1^4 x^{-\frac{1}{2}} \ln x\,\mathrm{d}x$, giving your answer in an exact form. **[5]**

4 By sketching the graphs of $y = |2x+1|$ and $y = -x$ on the same axes, show that the equation $|2x+1| = -x$ has two roots. Find these roots. **[4]**

5 The volume V m^3 of a pile of grain of height h metres is modelled by the equation

$$V = 4\sqrt{h^3+1} - 4.$$

(i) Find $\frac{\mathrm{d}V}{\mathrm{d}h}$ when $h = 2$. **[4]**

At a certain time, the height of the pile is 2 metres, and grain is being added so that the volume is increasing at a rate of 0.4 m^3 per minute.

(ii) Find the rate at which the height is increasing at this time. **[3]**

6 Fig. 6 shows part of the curve sin $2y = x - 1$. P is the point with coordinates $(1.5, \frac{1}{12}\pi)$ on the curve.

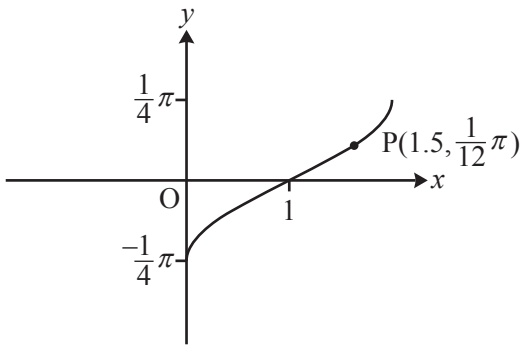

Fig. 6

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y.

Hence find the exact gradient of the curve sin $2y = x - 1$ at the point P. **[4]**

The part of the curve shown is the image of the curve y = arcsin x under a sequence of two geometrical transformations.

(ii) Find y in terms of x for the curve sin $2y = x - 1$.

Hence describe fully the sequence of transformations. **[4]**

7 You are given that n is a positive integer.

By expressing $x^{2n} - 1$ as a product of two factors, prove that $2^{2n} - 1$ is divisible by 3. **[4]**

Section B (36 marks)

8 Fig. 8 shows the curve $y = \dfrac{x}{\sqrt{x+4}}$ and the line $x = 5$. The curve has an asymptote l.

The tangent to the curve at the origin O crosses the line l at P and the line $x = 5$ at Q.

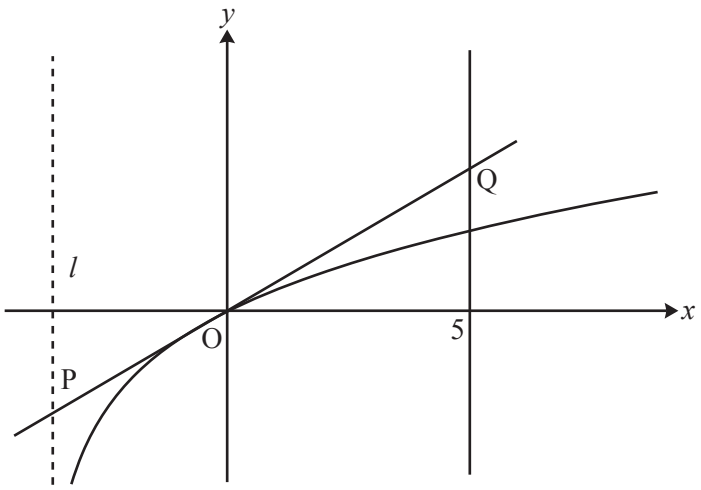

Fig. 8

(i) Show that for this curve $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x+8}{2(x+4)^{\frac{3}{2}}}$. **[5]**

(ii) Find the coordinates of the point P. **[4]**

(iii) Using integration by substitution, find the exact area of the region enclosed by the curve, the tangent OQ and the line $x = 5$. **[9]**

9 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \mathrm{e}^{2x} + k\,\mathrm{e}^{-2x}$ and k is a constant greater than 1.

The curve crosses the y-axis at P and has a turning point Q.

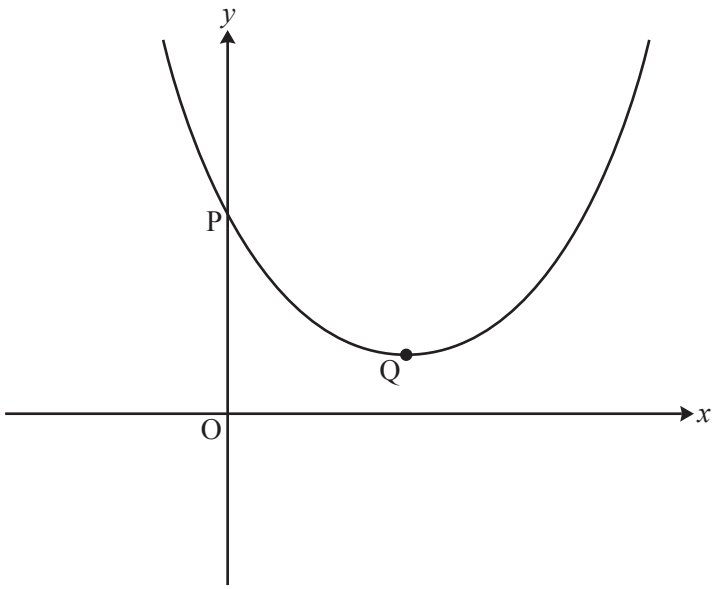

Fig. 9

(i) Find the y-coordinate of P in terms of k. **[1]**

(ii) Show that the x-coordinate of Q is $\frac{1}{4}\ln k$, and find the y-coordinate in its simplest form. **[5]**

(iii) Find, in terms of k, the area of the region enclosed by the curve, the x-axis, the y-axis and the line $x = \frac{1}{2}\ln k$. Give your answer in the form $ak + b$. **[4]**

The function $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = \mathrm{f}(x + \frac{1}{4}\ln k)$.

(iv) (*A*) Show that $\mathrm{g}(x) = \sqrt{k}\,(\mathrm{e}^{2x} + \mathrm{e}^{-2x})$. **[3]**

(*B*) Hence show that $\mathrm{g}(x)$ is an even function. **[2]**

(*C*) Deduce, with reasons, a geometrical property of the curve $y = \mathrm{f}(x)$. **[3]**

END OF QUESTION PAPER