



Oxford Cambridge and RSA

## Friday 24 June 2016 – Morning

### A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

#### QUESTION PAPER



Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

#### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

#### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

#### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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## Section A (36 marks)

1 Express  $\cos \theta - 3 \sin \theta$  in the form  $R \cos(\theta + \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{\pi}{2}$ .

Hence show that the equation  $\cos \theta - 3 \sin \theta = 4$  has no solution. [6]

2 Given that  $\left(1 + \frac{x}{p}\right)^q = 1 - x + \frac{3}{4}x^2 + \dots$ , find  $p$  and  $q$ , and state the set of values of  $x$  for which the expansion is valid. [7]

3 Fig. 3 shows the curve  $y = x^4$  and the line  $y = 4$ .

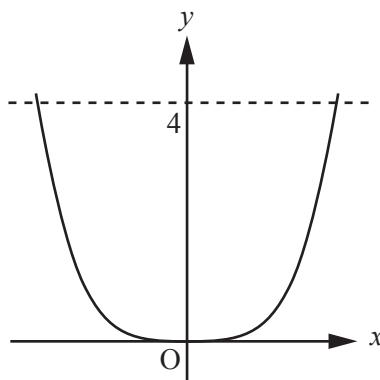


Fig. 3

The finite region enclosed by the curve and the line is rotated through  $180^\circ$  about the  $y$ -axis. Find the exact volume of revolution generated. [4]

4 Solve the equation  $2 \sin 2\theta = 1 + \cos 2\theta$  for  $0^\circ \leq \theta \leq 180^\circ$ . [5]

5 In Fig. 5, triangles ABC, ACD and ADE are all right-angled, and angles BAC, CAD and DAE are all  $\theta$ .

$AB = x$  and  $AE = 2x$ .

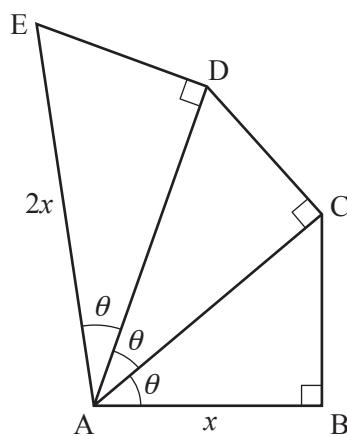


Fig. 5

(i) Show that  $\sec^3 \theta = 2$ . [3]

(ii) Hence show the ratio of lengths ED to CB is  $2^{\frac{2}{3}} : 1$ . [4]

6 P is a general point on the curve with parametric equations  $x = 2t$ ,  $y = \frac{2}{t}$ . This is shown in Fig. 6. The tangent at P intersects the  $x$ - and  $y$ -axes at the points Q and R respectively.

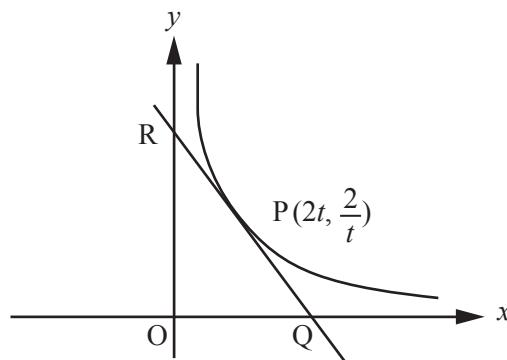


Fig. 6

Show that the area of the triangle OQR, where O is the origin, is independent of  $t$ .

[7]

## Section B (36 marks)

7 Fig. 7 shows a cuboid OABCDEFG with coordinates as shown. The point P has coordinates (4, 2, 0).

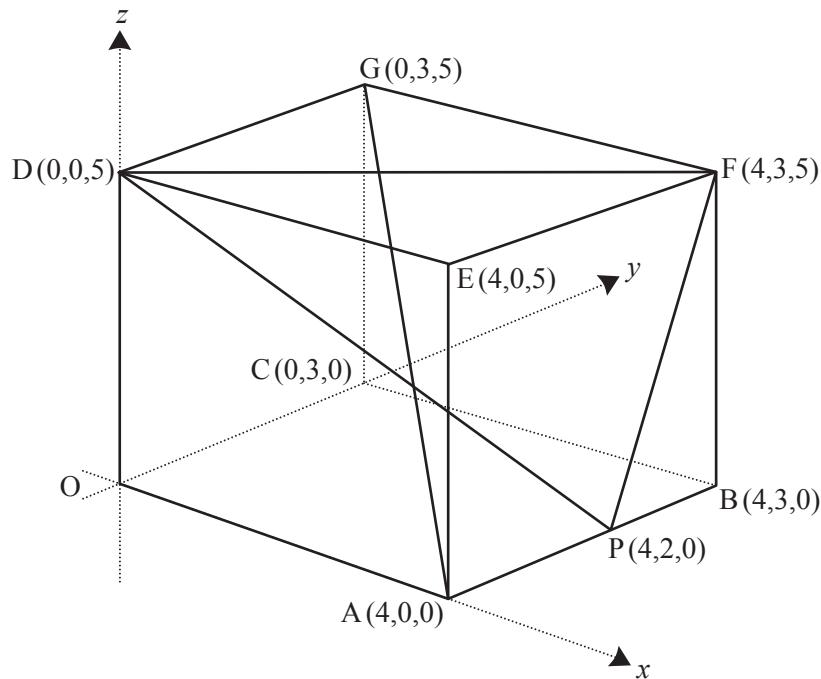


Fig. 7

(i) Find the length of the diagonal AG. [2]

(ii) Show that the vector  $\mathbf{n} = 15\mathbf{i} - 20\mathbf{j} + 4\mathbf{k}$  is normal to the plane DPF. Hence find the cartesian equation of this plane. [6]

The diagonal AG intersects the plane DPF at Q.

(iii) Write down a vector equation of the line AG. Hence find the coordinates of the point Q, and the ratio AQ:QG. [6]

(iv) Find the acute angle between the line AG and the plane DPF. [4]

8 (i) Show that  $\frac{1}{2+x} + \frac{1}{2-x} = \frac{4}{(2+x)(2-x)}$ . [1]

In a chemical reaction, the time  $t$  minutes taken for a mass  $x$  mg of a substance to be produced is modelled by the equation

$$t = \ln\left(\frac{2+x}{2-x}\right).$$

(ii) Show that when  $t = 0, x = 0$ . [2]

(iii) Show that the rate of change of  $x$  is proportional to the product of  $(2 + x)$  and  $(2 - x)$ , and find the constant of proportionality. [4]

(iv) Show that  $x = \frac{2(1 - e^{-t})}{1 + e^{-t}}$ .

Hence determine the long-term mass of the substance predicted by this model. [4]

In another chemical reaction, the mass  $x$  mg at time  $t$  minutes is modelled by the differential equation

$$\frac{dx}{dt} = k(2+x)(2-x)e^{-t},$$

where  $k$  is a positive constant, and  $x = 0$  when  $t = 0$ .

(v) Show by integration that, for this reaction,  $\ln\left(\frac{2+x}{2-x}\right) = 4k(1 - e^{-t})$ . [5]

(vi) Given that the long-term mass of this substance is 1.85 mg, find the value of  $k$ . [2]

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