OCR
Oxford Cambridge and RSA

Friday 20 May 2016 – Morning

AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4755/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 8 & -2 \\ p & 1 \end{pmatrix}$, where $p \neq -4$.

(i) Find the inverse of **M** in terms of p. **[2]**

(ii)

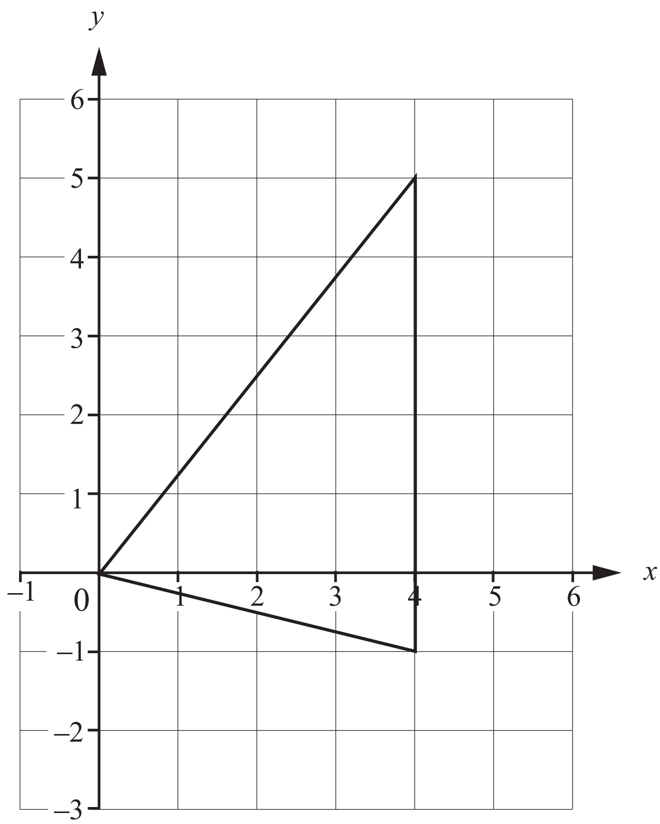

Fig. 1

The triangle shown in Fig. 1 undergoes the transformation represented by the matrix $\begin{pmatrix} 8 & -2 \\ 3 & 1 \end{pmatrix}$. Find the area of the image of the triangle following this transformation. **[2]**

2 The complex number z_1 is $2-5\mathrm{j}$ and the complex number z_2 is $(a-1)+(2-b)\mathrm{j}$, where a and b are real.

(i) Express $\dfrac{z_1^*}{z_1}$ in the form $x+y\mathrm{j}$, giving x and y in exact form. You must show clearly how you obtain your answer. **[4]**

(ii) Given that $\dfrac{z_1^*}{z_1} = z_2$, find the exact values of a and b. **[2]**

3 You are given that $\mathbf{A} = \begin{pmatrix} \lambda & 6 & -4 \\ 2 & 5 & -1 \\ -1 & 4 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -19 & 34 & -14 \\ 5 & -5 & 5 \\ -13 & 18 & -3 \end{pmatrix}$ and $\mathbf{AB} = \mu\mathbf{I}$, where $\mathbf{I}$ is the 3×3 identity matrix.

(i) Find the values of λ and μ. **[4]**

(ii) Hence find $\mathbf{B}^{-1}$. **[2]**

4 **(i)** Use standard series to show that

$$\sum_{r=1}^{n} r^2(2r-p) = \frac{1}{6}n(n+1)(3n^2+(3-2p)n-p),$$

where p is a constant. **[4]**

(ii) Given that the coefficients of n^3 and n^4 in the expression for $\sum_{r=1}^{n} r^2(2r-p)$ are equal, find the value of p. **[2]**

5 The loci C_1 and C_2 are given by $|z+3-4\mathrm{j}| = 5$ and $\arg(z+3-6\mathrm{j}) = \frac{1}{2}\pi$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. **[5]**

(ii) Write down the complex number represented by the point of intersection of C_1 and C_2. **[1]**

(iii) Indicate, by shading on your sketch, the region satisfying

$$|z+3-4\mathrm{j}| \geqslant 5 \quad \text{and} \quad \frac{1}{2}\pi \leqslant \arg(z+3-6\mathrm{j}) \leqslant \frac{3}{4}\pi.$$ **[2]**

6 A sequence is defined by $u_1 = 8$ and $u_{n+1} = 3u_n + 2n + 5$. Prove by induction that $u_n = 4(3^n) - n - 3$. **[6]**

Section B (36 marks)

7 The function $f(z) = 2z^4 - 9z^3 + Az^2 + Bz - 26$ has real coefficients. The equation $f(z) = 0$ has two real roots, α and β, where $\alpha > \beta$, and two complex roots, γ and δ, where $\gamma = 3 + 2\mathrm{j}$.

(i) Show that $\alpha + \beta = -\frac{3}{2}$ and find the value of $\alpha\beta$. **[5]**

(ii) Hence find the two real roots α and β. **[3]**

(iii) Find the values of A and B. **[3]**

(iv) Write down the roots of the equation $f\left(\frac{w}{\mathrm{j}}\right) = 0$. **[2]**

8 A curve has equation $y = \dfrac{3x^2 - 9}{x^2 + 3x - 4}$.

(i) Find the equations of the two vertical asymptotes and the one horizontal asymptote of this curve. **[3]**

(ii) State, with justification, how the curve approaches the horizontal asymptote for large positive and large negative values of x. **[3]**

(iii) Sketch the curve. **[3]**

(iv) Solve the inequality $\dfrac{3x^2 - 9}{x^2 + 3x - 4} \geqslant 0$. **[3]**

9 You are given that $\dfrac{3}{4(2r-1)} - \dfrac{1}{2r+1} + \dfrac{1}{4(2r+3)} = \dfrac{2r+5}{(2r-1)(2r+1)(2r+3)}$.

(i) Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{2r+5}{(2r-1)(2r+1)(2r+3)} = \frac{2}{3} - \frac{3}{4(2n+1)} + \frac{1}{4(2n+3)}.$$ **[6]**

(ii) Write down the limit to which $\displaystyle\sum_{r=1}^{n} \frac{2r+5}{(2r-1)(2r+1)(2r+3)}$ converges as n tends to infinity. **[1]**

(iii) Find the sum of the finite series

$$\frac{45}{39 \times 41 \times 43} + \frac{47}{41 \times 43 \times 45} + \frac{49}{43 \times 45 \times 47} + \dots + \frac{105}{99 \times 101 \times 103},$$

giving your answer to 3 significant figures. **[4]**

END OF QUESTION PAPER