

OCR

Oxford Cambridge and RSA

Friday 20 May 2016 – Morning

AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

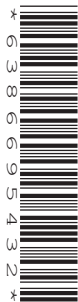
OCR supplied materials:

- Printed Answer Book 4755/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

- 1 The matrix \mathbf{M} is given by $\mathbf{M} = \begin{pmatrix} 8 & -2 \\ p & 1 \end{pmatrix}$, where $p \neq -4$.

(i) Find the inverse of \mathbf{M} in terms of p .

[2]

(ii)

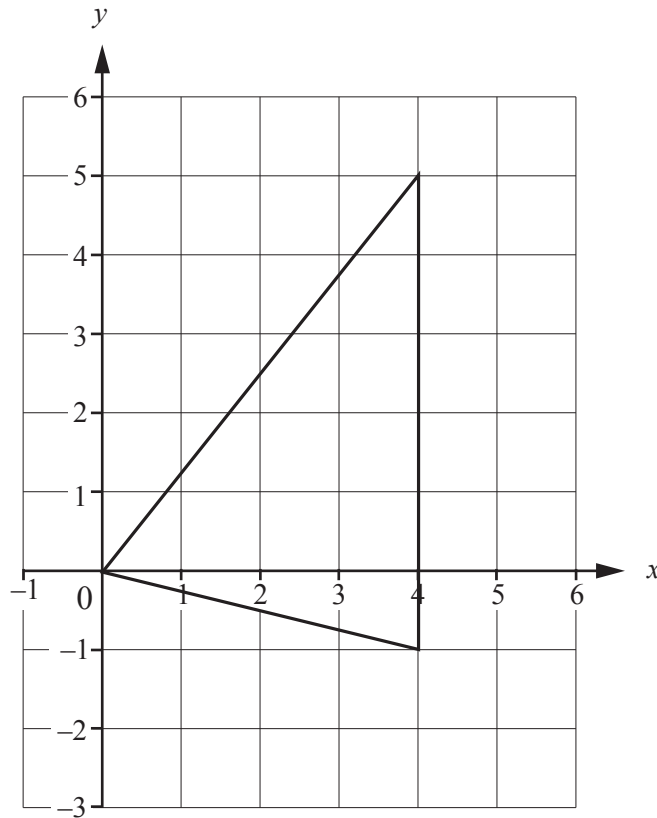


Fig. 1

The triangle shown in Fig. 1 undergoes the transformation represented by the matrix $\begin{pmatrix} 8 & -2 \\ 3 & 1 \end{pmatrix}$. Find the area of the image of the triangle following this transformation. [2]

- 2 The complex number z_1 is $2 - 5j$ and the complex number z_2 is $(a - 1) + (2 - b)j$, where a and b are real.

(i) Express $\frac{z_1^*}{z_1}$ in the form $x + yj$, giving x and y in exact form. You must show clearly how you obtain your answer. [4]

(ii) Given that $\frac{z_1^*}{z_1} = z_2$, find the exact values of a and b . [2]

- 3 You are given that $\mathbf{A} = \begin{pmatrix} \lambda & 6 & -4 \\ 2 & 5 & -1 \\ -1 & 4 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -19 & 34 & -14 \\ 5 & -5 & 5 \\ -13 & 18 & -3 \end{pmatrix}$ and $\mathbf{AB} = \mu \mathbf{I}$, where \mathbf{I} is the 3×3 identity matrix.

(i) Find the values of λ and μ . [4]

(ii) Hence find \mathbf{B}^{-1} . [2]

- 4 (i) Use standard series to show that

$$\sum_{r=1}^n r^2(2r-p) = \frac{1}{6}n(n+1)(3n^2 + (3-2p)n - p),$$

where p is a constant. [4]

(ii) Given that the coefficients of n^3 and n^4 in the expression for $\sum_{r=1}^n r^2(2r-p)$ are equal, find the value of p . [2]

- 5 The loci C_1 and C_2 are given by $|z+3-4j|=5$ and $\arg(z+3-6j) = \frac{1}{2}\pi$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2 . [5]

(ii) Write down the complex number represented by the point of intersection of C_1 and C_2 . [1]

(iii) Indicate, by shading on your sketch, the region satisfying

$$|z+3-4j| \geq 5 \quad \text{and} \quad \frac{1}{2}\pi \leq \arg(z+3-6j) \leq \frac{3}{4}\pi. \quad [2]$$

- 6 A sequence is defined by $u_1 = 8$ and $u_{n+1} = 3u_n + 2n + 5$. Prove by induction that $u_n = 4(3^n) - n - 3$. [6]

Section B (36 marks)

- 7 The function $f(z) = 2z^4 - 9z^3 + Az^2 + Bz - 26$ has real coefficients. The equation $f(z) = 0$ has two real roots, α and β , where $\alpha > \beta$, and two complex roots, γ and δ , where $\gamma = 3 + 2j$.

(i) Show that $\alpha + \beta = -\frac{3}{2}$ and find the value of $\alpha\beta$. [5]

(ii) Hence find the two real roots α and β . [3]

(iii) Find the values of A and B . [3]

(iv) Write down the roots of the equation $f\left(\frac{w}{j}\right) = 0$. [2]

- 8 A curve has equation $y = \frac{3x^2 - 9}{x^2 + 3x - 4}$.

(i) Find the equations of the two vertical asymptotes and the one horizontal asymptote of this curve. [3]

(ii) State, with justification, how the curve approaches the horizontal asymptote for large positive and large negative values of x . [3]

(iii) Sketch the curve. [3]

(iv) Solve the inequality $\frac{3x^2 - 9}{x^2 + 3x - 4} \geq 0$. [3]

- 9 You are given that $\frac{3}{4(2r-1)} - \frac{1}{2r+1} + \frac{1}{4(2r+3)} = \frac{2r+5}{(2r-1)(2r+1)(2r+3)}$.

(i) Use the method of differences to show that

$$\sum_{r=1}^n \frac{2r+5}{(2r-1)(2r+1)(2r+3)} = \frac{2}{3} - \frac{3}{4(2n+1)} + \frac{1}{4(2n+3)}. \quad [6]$$

(ii) Write down the limit to which $\sum_{r=1}^n \frac{2r+5}{(2r-1)(2r+1)(2r+3)}$ converges as n tends to infinity. [1]

(iii) Find the sum of the finite series

$$\frac{45}{39 \times 41 \times 43} + \frac{47}{41 \times 43 \times 45} + \frac{49}{43 \times 45 \times 47} + \dots + \frac{105}{99 \times 101 \times 103},$$

giving your answer to 3 significant figures. [4]

END OF QUESTION PAPER

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series. If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.