

OCR

Oxford Cambridge and RSA

Monday 27 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Section A (54 marks)

- 1 (a) (i) Given that $f(x) = \arctan x$, write down an expression for $f'(x)$. Assuming that x is small, use a binomial expansion to express $f'(x)$ in ascending powers of x as far as the term in x^4 . [3]

- (ii) Hence express $\arctan x$ in ascending powers of x as far as the term in x^5 . [3]

- (b) Find, in exact form, the value of the following integral.

$$\int_0^{\frac{3}{4}} \frac{1}{\sqrt{3-4x^2}} dx \quad [5]$$

- (c) A curve has polar equation $r = \frac{a}{\sqrt{\theta}}$ where $a > 0$.

- (i) Sketch the curve for $\frac{\pi}{4} \leq \theta \leq 2\pi$. [2]

- (ii) State what happens to r as θ tends to zero. [1]

- (iii) Find the area of the region enclosed by the part of the curve sketched in part (i) and the lines $\theta = \frac{\pi}{4}$ and $\theta = 2\pi$. Give your answer in an exact simplified form. [4]

- 2 (a) (i) Express $2 \sin \frac{1}{2}\theta (\sin \frac{1}{2}\theta - j \cos \frac{1}{2}\theta)$ in terms of z where $z = \cos \theta + j \sin \theta$. [3]

- (ii) The series C and S are defined as follows.

$$C = 1 - \binom{n}{1} \cos \theta + \binom{n}{2} \cos 2\theta - \dots + (-1)^n \binom{n}{n} \cos n\theta$$

$$S = -\binom{n}{1} \sin \theta + \binom{n}{2} \sin 2\theta - \dots + (-1)^n \binom{n}{n} \sin n\theta$$

Show that

$$C + jS = \left\{ -2j \sin \frac{1}{2}\theta (\cos \frac{1}{2}\theta + j \sin \frac{1}{2}\theta) \right\}^n.$$

Hence show that, for even values of n ,

$$\frac{C}{S} = \cot\left(\frac{1}{2}n\theta\right). \quad [8]$$

- (b) Write the complex number $z = \sqrt{6} + j\sqrt{2}$ in the form $re^{j\theta}$, expressing r and θ as simply as possible.

Hence find the cube roots of z in the form $re^{j\theta}$.

Show the points representing z and its cube roots on an Argand diagram. [7]

- 3 (i) Find the eigenvalues and eigenvectors of the matrix \mathbf{M} , where

$$\mathbf{M} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}.$$

Hence express \mathbf{M} in the form \mathbf{PDP}^{-1} where \mathbf{D} is a diagonal matrix. [8]

- (ii) Write down an equation for \mathbf{M}^n in terms of the matrices \mathbf{P} and \mathbf{D} .

Hence obtain expressions for the elements of \mathbf{M}^n .

Show that \mathbf{M}^n tends to a limit as n tends to infinity. Find that limit. [6]

- (iii) Express \mathbf{M}^{-1} in terms of the matrices \mathbf{P} and \mathbf{D} . Hence determine whether or not $(\mathbf{M}^{-1})^n$ tends to a limit as n tends to infinity. [4]

Section B (18 marks)

- 4 (i) Given that $y = \cosh x$, use the definition of $\cosh x$ in terms of exponential functions to prove that

$$x = \pm \ln(y + \sqrt{y^2 - 1}).$$
 [5]

- (ii) Solve the equation

$$\cosh x + \cosh 2x = 5,$$

giving the roots in an exact logarithmic form. [5]

- (iii) Sketch the curve with equation $y = \cosh x + \cosh 2x$. Show on your sketch the line $y = 5$.

Find the area of the finite region bounded by the curve and the line $y = 5$. Give your answer in an exact form that does not involve hyperbolic functions. [8]

END OF QUESTION PAPER

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