

OCR

Oxford Cambridge and RSA

Friday 17 June 2016 – Afternoon

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ ms}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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1 The differential equation

$$4\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 3x = f(t)$$

is to be solved.

Consider first the case where $f(t) = 4e^{0.5t}$.

(i) Find the general solution for x . [7]

(ii) Given that when $t = 0$, $x = 6$ and $\frac{dx}{dt} = -4$, find the particular solution for x . [4]

(iii) Given that x has a minimum value, find the value of t for which this minimum occurs. [4]

Now consider the case where $f(t) = 4e^{-0.5t}$.

(iv) Given that when $t = 0$, $x = 6$ and $\frac{dx}{dt} = -4$, find the particular solution for x . [9]

2 (a) The differential equation

$$x\frac{dy}{dx} - 3y = x^5 \cos x$$

is to be solved.

(i) Find the general solution for y in terms of x . [8]

(ii) Find the particular solution for which $y = 0$ when $x = \frac{1}{2}\pi$. [2]

(b) Now consider the differential equation $\sec x \frac{dy}{dx} - 3y^2 = 0$.

(i) Find the particular solution for which $y = 1$ when $x = \frac{1}{2}\pi$. [7]

(ii) Show that the maximum value of y is 1. [2]

(c) Now consider the differential equation $\sec x \frac{dy}{dx} - 3y^2 = x$, where $y = 0$ when $x = 1$.

This is to be solved numerically using Euler's method. The algorithm is given by

$$x_{r+1} = x_r + h, \quad y_{r+1} = y_r + hy'_r.$$

Use a step length of 0.01 to estimate y when $x = 1.02$. [5]

- 3 A parachutist of mass 90 kg falls vertically from a stationary helicopter. When he is x m below the helicopter, his velocity is $v \text{ m s}^{-1}$. The forces acting on the parachutist are his weight and a resistive force of magnitude $0.36v^2 \text{ N}$.

(i) Show that his motion can be modelled by the differential equation

$$v \frac{dv}{dx} = 9.8 - 0.004v^2. \quad [2]$$

(ii) Solve this differential equation to show that

$$v^2 = 2450(1 - e^{-0.008x}). \quad [6]$$

(iii) Sketch the graph of v against x . [2]

(iv) Find how far the parachutist has fallen when his speed has reached 48 m s^{-1} . [2]

The parachutist opens his parachute when his speed is 48 m s^{-1} . The forces acting on him now are his weight and a resistive force of magnitude $72v \text{ N}$.

(v) Find an expression for v in terms of t , where t is the time in seconds that has elapsed since the parachute was opened. [8]

(vi) Find the distance that the parachutist falls during the first 5 seconds after his parachute has opened. [4]

- 4 The simultaneous differential equations

$$\frac{dx}{dt} = x - y + 3 \cos t$$

$$\frac{dy}{dt} = 5x - y - 12 \sin t$$

are to be solved for $t \geq 0$, where t denotes time.

(i) Eliminate y to obtain a second order differential equation for x in terms of t . Hence find the general solution for x . [12]

(ii) Find the corresponding general solution for y . [3]

When $t = 0$, $y = 0$ and $\frac{dy}{dt} = 5$.

(iii) Find the particular solutions for x and y . [4]

(iv) Find the time that elapses between the first two occasions on which $x = y$. [5]

END OF QUESTION PAPER

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