OCR
Oxford Cambridge and RSA

Wednesday 29 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4773/01 Decision Mathematics Computation

Candidates answer on the Answer Booklet.

OCR supplied materials:
- 12 page Answer Booklet (OCR12) (sent with general stationery)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Duration: 2 hours 30 minutes

* 4 7 7 3 0 1 *

* 6 3 8 3 2 0 1 4 7 6 *

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.
- Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

1 A weed in the lawn sends out seeds and then dies. Some of those seeds germinate and produce new weeds (the first generation). Each of the new weeds sends out seeds and then dies. Those seeds give rise to the second generation, and so on.

The number of new weeds generated by each weed is a random variable specified by the probability distribution given in the table.

Number of new weeds	0	1	2	3
Probability	0.2	0.4	0.3	0.1

(i) Why can there be up to 27 weeds in generation 3, and what is the probability of 27 occurring? **[2]**

(ii) Build a spreadsheet simulation of this process for generations 1, 2 and 3. **[6]**

(iii) Use your simulation model to estimate the probability of extinction by generation 2, i.e. the probability that there are no weeds in generation 2. Explain how you made your estimate. **[4]**

(iv) Use your simulation model to estimate the probabilities of having 0, 1, 2, 3 or >3 weeds in generation 3. Say how many repetitions of your simulation you used, and why. **[5]**

(v) Use your answers to parts **(iii)** and **(iv)** to estimate the probability of extinction **at** generation 3. **[1]**

2 Basil is planning his new garden. He has identified eight places in which he can locate shrubs, some in the ground and some in pots of various sizes. He has six shrubs to plant. The table shows which shrubs can be planted in which locations.

		Location							
		1	2	3	4	5	6	7	8
Shrub	1	x					x		
	2	x				x	x	x	
	3			x	x		x		x
	4	x						x	
	5		x		x	x			x
	6		x	x				x	

(i) Draw a bipartite graph showing the information in the table. **[1]**

(ii) Draw a bipartite graph showing the incomplete matching (S2, L1), (S3, L8), (S5, L2), (S6, L7). **[1]**

(iii) Identify a path connecting S1 with L3 which alternates edges not in the graph in part **(ii)** with edges that are in part **(ii)**. Hence identify an improved matching and draw it on a third bipartite graph. **[4]**

(iv) Find a maximal matching. **[1]**

(v) Formulate an LP to find a maximal matching. **[4]**

(vi) Run your LP and interpret the results. **[2]**

The costs of planting shrubs vary by location because of the costs of pots, compost, irrigation, etc. The costs are shown in the table.

		Location							
		1	2	3	4	5	6	7	8
Shrub	1	10					10		
	2	10				20	10	5	
	3			10	20		10		5
	4	12						7	
	5		7		20	20			7
	6		5	10				5	

(vii) Formulate, run and interpret an LP to find a minimum cost maximal allocation. **[5]**

3 A distribution company is planning to expand its operations into a new locality. It has identified potential customers, locations for potential depots and the annual costs of supplying each customer from each depot. These costs are shown in the table.

		Customer						
		1	2	3	4	5	6	7
Depot	1	2500		3000	1600	5200	4700	3750
	2	3450	6700	3000	2250		5450	2100
	3			2400	1500	4300	3800	1750
	4	2100	5700	4800	2390			2560
	5	5200	5600		3430		6300	3400

In addition, annual costs of renting and running the potential depots are given below.

Depot	1	2	3	4	5
Annual cost	22000	30000	28000	25000	22000

(i) Formulate, run and interpret an integer program to find which depots to establish and which customers to service from which depot. **[10]**

(ii) To what must the annual cost of depot 2 be reduced to make it worth using? Show that your reduced cost is critical and give the best distribution arrangement if depot 2 is used. **[4]**

(iii) Assume that the annual cost of depot 2 has not been reduced. To what must the cost of servicing customer 1 from depot 1 rise for it not to be best to serve customer 1 from depot 1? Show that your increased cost is critical and give the best distribution arrangement if customer 1 is not served from depot 1. **[4]**

4 Ulrike wants to expand her milk delivery business. Ulrike had 2000 customers last week, and has 2100 customers this week.

She argues that, even if she loses 1% of her existing customers each week, as long as her marketing increases the number of customers by 1% more than the previous week's increase, then her total number of customers will increase. Thus next week she can afford to lose 21 customers (1% of 2100) if her marketing gains her 101 new customers (101% of (2100–2000)).

(i) Letting u_n be the number of customers that Ulrike has in week n, give a recurrence relation for u_{n+2} in terms of u_{n+1} and u_n that models Ulrike's argument. Check that with $u_0 = 2000$ and $u_1 = 2100$, your relationship gives $u_2 = 2180$. **[3]**

(ii) Show that the auxiliary equation for your recurrence relation is $\lambda^2 - (\alpha + \beta)\lambda + \beta = 0$, where $\alpha = 0.99$ and $\beta = 1.01$. Show that the equation does not have real roots. **[3]**

(iii) The auxiliary equation for u_n does not have real roots. What does this tell you about the behaviour of u_n? **[1]**

(iv) Construct a spreadsheet for u_n and describe what happens. **[5]**

There are two ways in which the recurrence relation for u_n cannot reflect reality. One is that the values that u_n can take in reality are non-negative whole numbers. The other is that Ulrike would not wish to argue that if her customer numbers dropped from one week to the next, then her marketing would result in them dropping by 1% more in the following week.

(v) Amend your spreadsheet as follows:

- if the previous week showed a decrease in the number of customers, there are no new customers to add
- if the previous week showed an increase in the number of customers, the number of new customers is 101% of that increase
- the final computed number of customers is given to the nearest integer. **[2]**

Ulrike needs convincing that, in following this model, the only realistic way she can save her business in the long run is to reduce the percentage of her existing customers that she loses each week.

(vi) Use your spreadsheet from part **(v)** to illustrate this argument. **[2]**

(vii) Use the auxiliary equation $\lambda^2 - (0.99 + \beta)\lambda + \beta = 0$ to make the argument by showing that, with a 1% drop in existing customers, β needs to be at least 1.21 to avoid oscillation. **[2]**

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