OCR
Oxford Cambridge and RSA

Wednesday 29 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4777/01 Numerical Computation

Candidates answer on the Answer Booklet.

OCR supplied materials:
- 12 page Answer Booklet (OCR12) (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)
- Graph paper

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Duration: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.
- Do **not** write in the bar codes.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program and suitable printing facilities throughout the examination.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- In each of the questions you are required to write spreadsheet routines to carry out various numerical analysis processes.
- You will not receive credit for using any numerical analysis functions which are provided within the spreadsheet. For example, many spreadsheets provide a solver routine; you will not receive credit for using this routine when asked to write your own procedure for solving an equation.
  You may use the following built-in mathematical functions: square root, sin, cos, tan, arcsin, arccos, arctan, ln, exp.
- For each question you attempt, you should submit print-outs showing the spreadsheet routine you have written and the output it generates. It will be necessary to print out the formulae in the cells as well as the values in the cells.
  You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 **(i)** The equation $f(x) = 0$ has a root α. The equation is rearranged to $x = g(x)$, and the corresponding iterative formula gives the sequence $x_0, x_1, x_2, \ldots$.

Given that $x_{r+1} - \alpha \approx k(x_r - \alpha)$ for some constant k, where $k \neq 1$, show that

$$\alpha \approx \frac{kx_0 - x_1}{k-1}, \qquad \text{where} \qquad k \approx \frac{x_2 - x_1}{x_1 - x_0}.$$ **[3]**

(ii) Show graphically that the equation

$$e^x + e^{-x} = 5\sin x, \qquad (*)$$

where x is in radians, has exactly two solutions. **[4]**

(iii) Show numerically that the iterative formula

$$x_{r+1} = \ln(5\sin x_r - e^{-x_r})$$

converges to the larger root but does not converge to the smaller root.

Show that the smaller root may be found using the technique in part **(i)** with this iterative formula. **[11]**

(iv) Equation (*) is a special case of the equation

$$e^x + e^{-x} = c\sin x. \qquad (**)$$

Obtain the value of c, correct to 3 significant figures, for which the difference between the two roots of (**) is 1. **[6]**

2 The Gaussian 3-point integration formula has the form

$$\int_{-h}^{h} \mathrm{f}(x)\,\mathrm{d}x = a\mathrm{f}(-\alpha) + b\mathrm{f}(0) + a\mathrm{f}(\alpha).$$

(i) Obtain the equations that determine a, b and α. Verify that these equations are satisfied by

$$\alpha = \sqrt{\frac{3}{5}}\,h, \quad a = \frac{5}{9}h, \quad b = \frac{8}{9}h.$$ **[8]**

(ii) By considering $\mathrm{f}(x) = x^5$ and $\mathrm{f}(x) = x^6$, determine the orders of the local and global errors in the Gaussian 3-point rule. **[4]**

(iii) The integral

$$I = \int_0^{\pi} \exp(k\sin x)\,\mathrm{d}x$$

(where exp denotes the exponential function) is to be evaluated using the Gaussian 3-point rule.

Find I when (A) $k = 1$,

(B) $k = -1$. **[10]**

(iv) Find, correct to 2 significant figures, the value of k for which $I = 3$. **[2]**

3 The differential equation

$$xy'' + y' = x(1 - y^2),$$

with initial conditions $y = 1$ and $y' = 1$ when $x = 1$, is to be solved using finite difference methods.

(i) Show that, in the usual notation, central difference approximations give the following equations.

$$y_{r+1} = \frac{2h^2 x_r(1 - y_r^2) + 4x_r y_r - (2x_r - h)y_{r-1}}{2x_r + h}$$

$$y_1 = 1 + h - \frac{1}{2}h^2$$

[8]

(ii) Obtain a graph of the solution curve from $x = 1$ to $x = 10$. **[8]**

(iii) Determine, correct to 2 decimal places, the coordinates of the first maximum on the curve. **[4]**

(iv) Confirm, by considering estimates of y when $x = 3$, that the convergence is 2nd order. **[4]**

4 **(i)** Show that the Gauss-Seidel method applied directly to the following system of equations

$$\begin{pmatrix} 1 & 4 & 0 & 2 \\ k & -1 & 1 & -2 \\ 0 & 1 & -3 & 6 \\ 2 & 1 & 4 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

does not converge when $k = 5$.

Explain this in terms of diagonal dominance. **[8]**

(ii) With k still equal to 5, show that diagonal dominance can be achieved by re-ordering the system of equations. Hence obtain the solution using the Gauss-Seidel method. Verify your solution. **[8]**

(iii) Obtain the solutions for $k = 1, 2, \dots, 6$. Draw, on the same axes, graphs of x_1, x_2, x_3, x_4, against k. **[8]**

END OF QUESTION PAPER