OCR
Oxford Cambridge and RSA

Wednesday 29 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4798/01 Further Pure Mathematics with Technology (FPT)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4798/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software

Duration: Up to 2 hours

* 6 3 8 5 1 1 8 9 1 4 *

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a computer algebra system, a spreadsheet, a programming language and graph-plotting software throughout the examination.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 This question concerns the family of curves with parametric equations

$$x = \cos t, \quad y = \sin t - k\tan\frac{t}{2},$$

where k is a positive integer and $-\pi < t < \pi$.

(i) Sketch the curves for the cases $k = 1$, $k = 2$ and $k = 3$ and give the points of intersection with the axes.

Describe two common features of these three curves and one distinct feature for each of the cases $k = 1$ and $k = 2$. **[7]**

(ii) For the case $k = 1$, find, in cartesian form, the points on the curve where the tangent to the curve is parallel to the x-axis. **[4]**

(iii) For the case $k = 2$, confirm the feature of the curve at the point where $t = 0$ by investigating the gradient as $t \to 0$. **[4]**

(iv) For the case $k = 3$, show algebraically that there are no points on the curve where the tangent to the curve is parallel to the x-axis. **[3]**

(v) Sketch the polar curve

$$r = \frac{\cos 2\theta}{\cos\theta}.$$

Show algebraically that the parametric equations

$$x = \cos t, \quad y = \sin t - \tan\frac{t}{2}$$

represent this polar curve.

[6]

2 **(i)** Find, in the form $x + \mathrm{i}y$, the values of $\sinh z$ for $z = \ln 2 + k\mathrm{i}$ where $k = -3, -2, -1, 0, 1, 2, 3$.

Sketch the points representing these values on an Argand diagram.

Show that the points in an Argand diagram representing $\sinh(\ln 2 + k\mathrm{i})$, where $k \in \mathbb{R}$, lie on an ellipse with equation of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are to be determined. **[8]**

(ii) F_1 and F_2 are the points representing the roots of the equation $z^2 + 1 = 0$ on an Argand diagram. The points A and B on the ellipse found in part **(i)** have coordinates $(a, 0)$ and $(0, b)$ respectively.

Show that $F_1A + F_2A = F_1B + F_2B$ where F_1A denotes the distance from F_1 to A. **[3]**

(iii) The function $f(z)$ has derivative $f'(z) = z^2 + 1$. Given that $f\left(\frac{5}{2}\mathrm{i}\right) = 0$, show that the points representing the roots of $f(z) = 0$ on an Argand diagram form an isosceles triangle and that the midpoints of the sides of this triangle lie on the ellipse in part **(i)**. **[8]**

(iv) Find the midpoints of the sides of the triangle formed by the points representing the roots of $z^3 + 3z + \frac{730\mathrm{i}}{27} = 0$ on an Argand diagram. Show that the complex numbers represented by these points can be written in the form $\sinh(\ln 3 + k\,\mathrm{i})$ where $k \in \mathbb{R}, -\pi < k < \pi$. **[5]**

3 This question concerns Gaussian integers z of the form $a + b\,\mathrm{i}$, where $a, b \in \mathbb{Z}$.

(i) Create a program that will find all the Gaussian integers z, in the form $a + b\,\mathrm{i}$, that are squares of Gaussian integers where $0 \leqslant a \leqslant 20$ and $0 \leqslant b \leqslant 20$.

You should write out your program in full and write down the Gaussian integers found. **[8]**

(ii) For the values of z found in part **(i)** for which $\mathrm{Re}(z) = 0$ and $\mathrm{Im}(z) > 0$, the complex numbers w are defined by $w^2 = z$ with $\mathrm{Re}(w) > 0$. Sketch the points representing w on an Argand diagram.

Show that $z = 2k^2\mathrm{i}$ for these values of z, where $k \in \mathbb{Z}$. **[3]**

(iii) Show that z is a positive real square of a Gaussian integer if, and only if, z is the square of a real integer. **[4]**

(iv) Show that if $a + b\mathrm{i}$ is the square of a Gaussian integer, where a and b are positive integers, then $a^2 + b^2 = c^2$ for some positive integer c. Show that the converse of this statement is not true. **[5]**

(v) Let z be a Gaussian integer of the form $v^2 + 1$ where v is a Gaussian integer. Find, in the form $a + b\,\mathrm{i}$, all the values of z for which $0 \leqslant a \leqslant 20$ and $0 \leqslant b \leqslant 20$. Indicate clearly the method you have used.

Show that $v^2 + 1$, where v is a Gaussian integer and $|v| > 2$, is never a Gaussian prime. **[4]**

END OF QUESTION PAPER