

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS  
AS GCE  
4776/01  
MATHEMATICS (MEI)  
Numerical Methods  
QUESTION PAPER  
TUESDAY 21 JUNE 2016:  
Morning  
DURATION: 1 hour 30 minutes  
plus your additional time allowance  
MODIFIED ENLARGED 24pt**

**Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.**

**OCR SUPPLIED MATERIALS:  
None**

**OTHER MATERIALS REQUIRED:  
Scientific or graphical calculator**

**READ INSTRUCTIONS OVERLEAF**



## **INSTRUCTIONS TO CANDIDATES**

**Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.**

**IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED.**

**Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**

**Use black ink. HB pencil may be used for graphs and diagrams only.**

**Read each question carefully. Make sure you know what you have to do before starting your answer.**

**Answer ALL the questions.**

**You are permitted to use a scientific or graphical calculator in this paper.**

**Final answers should be given to a degree of accuracy appropriate to the context.**

## **INFORMATION FOR CANDIDATES**

**The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.**

**You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.**

**The total number of marks for this paper is 72.**

**Any blank pages are indicated.**

## **INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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## SECTION A (36 marks)

1 The expression  $\sqrt{\frac{n}{n-1}}$  is sometimes approximated by 1 when  $n$  is large.

(i) Find the absolute and the relative error in this approximation when  $n = 40$ . [2]

(ii) Using trial and error or otherwise, find the smallest integer  $n$  for which the magnitude of the relative error is less than 1%. [2]

2 You are given that the equation

$$x^3 + x - 3 = 0$$

has a single real root  $\alpha$ , where  $1 < \alpha < 2$ .

(i) Use the Newton-Raphson method with  $x_0 = 1.5$  to find  $\alpha$  correct to 5 decimal places. [5]

(ii) By considering ratios of differences, show that the Newton-Raphson method is faster than first order. [3]

3 A function  $f(x)$  has the following values, correct to 5 decimal places. (The values of  $x$  are exact.)

$x$	0.5	1	1.5	2	2.5
$f(x)$	0.95885	0.84147	0.66500	0.45465	0.23939

(i) Obtain two Simpson's rule estimates of  $I = \int_{0.5}^{2.5} f(x) dx$ . [3]

(ii) State the order of Simpson's rule and hence estimate the value of  $I$  that would be obtained if  $f(x)$  were known at  $x = 0.75, 1.25, 1.75, 2.25$ . [4]

(iii) Give the value of  $I$  to the accuracy that is justified. [1]

- 4 (i) State the orders of accuracy of the forward difference and central difference formulae for numerical differentiation. Explain what this means in practice. [3]
- (ii) A function  $g(x)$  has the following values, correct to 5 decimal places. (The values of  $x$  are exact.)

$x$	-0.2	-0.1	0	0.1	0.2
$g(x)$	0.75560	0.87686	1	1.12314	1.24440

Obtain two estimates of  $g'(0)$  using the forward difference formula, and two estimates of  $g'(0)$  using the central difference formula.

Comment on your estimates. [5]

- 5 A function  $h(x)$  has values as shown in the table.

$x$	$h(x)$
0	1.35701
0.5	1.41333
1	1.38177
1.5	1.26431

- (i) Show, by means of a difference table, that  $h(x)$  can be well approximated by a quadratic. [3]
- (ii) Use Newton's forward difference interpolation formula with  $x_0 = 0$  to write down an expression for the quadratic approximation to  $h(x)$ . (You do not need to simplify this expression.) [3]
- (iii) Find the error in the quadratic approximation at  $x = 1.5$ . [2]

## SECTION B (36 marks)

6 (i) Show, by means of a sketch graph, that the equation

$$kx = 3^{-x}, \quad (*)$$

where  $k > 0$ , has exactly one root. [4]

(ii) Show numerically that the iterative formula

$$x_{r+1} = \frac{1}{k} 3^{-x_r}, \quad (**)$$

with  $x_0 = 1$ ,

(A) converges in the case  $k = 0.5$ ,

(B) diverges in the case  $k = 0.4$ .

Explain why it would NOT be a good idea to use (\*\*) in the case  $k = 0.5$ . [7]

(iii) Show that (\*) may be rearranged as

$$x = 0.5 \left( x + \frac{1}{k} 3^{-x} \right).$$

Use an iteration based on this rearrangement to find the root of (\*), correct to 4 decimal places, in the cases

(A)  $k = 0.5$ ,

(B)  $k = 0.4$ . [7]

**7 Let  $S_n$  be the sum of the first  $n$  terms in the series**

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \quad (*)$$

**It is known that  $S_n$  converges to a limit  $S$  as  $n$  tends to infinity. A spreadsheet is used to investigate the rate of convergence of  $S_n$  to  $S$ .**

**(i) The spreadsheet gives  $S_{1000} = 0.692\,647$ , hence  $0.692\,65$  to 5 decimal places.**

**Find  $S_{1001}$  and  $S_{1002}$  correct to 5 decimal places. Comment on the rate of convergence of  $S_n$ . [4]**

**(ii) Show, by combining adjacent terms, that (\*) may be written as**

$$\frac{1}{2} + \frac{1}{12} + \dots \quad (**)$$

**State the next two terms in this series. [3]**

**Let  $T_n$  be the sum of the first  $n$  terms of (\*\*). A spreadsheet is used to investigate the rate of convergence of  $T_n$ .**

**(iii) Explain why  $T_{500}$  will be  $0.692\,65$  correct to 5 decimal places.**

**Find  $T_{501}$  and  $T_{502}$  correct to 5 decimal places. Comment on the rate of convergence of  $T_n$ . [4]**

**An improved method for summing (\*\*) is to add a ‘correction term’ as follows.**

$$T_n + \frac{1}{4n+2} \quad (***)$$

**(iv) Evaluate (\*\*\*) correct to 5 decimal places for  $n = 500$  and  $n = 501$ .**

**Comment on your answers. [4]**

- (v) Discuss briefly what your answers to parts (i), (iii) and (iv) indicate about convergence when successive answers agree to a certain number of decimal places.**

**Explain which, if any, of the sums calculated you would regard as the value of  $S$  correct to 5 decimal places. [3]**

**END OF QUESTION PAPER**

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