OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4753/01

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

QUESTION PAPER

TUESDAY 21 JUNE 2016: Morning

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

None

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (NF/SW) 133031/1

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INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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SECTION A (36 marks)

1 Find the exact value of $\int_0^{\frac{1}{2}\pi} (1 + \cos \frac{1}{2}x)\,\mathrm{d}x$. [3]

2 The functions $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are defined by $\mathrm{f}(x) = \ln x$ and $\mathrm{g}(x) = 2 + \mathrm{e}^x$, for $x > 0$.

Find the exact value of x, given that $\mathrm{fg}(x) = 2x$. [5]

3 Find $\int_1^4 x^{-\frac{1}{2}} \ln x \,\mathrm{d}x$, giving your answer in an exact form. [5]

4 By sketching the graphs of $y = |2x + 1|$ and $y = -x$ on the same axes, show that the equation $|2x + 1| = -x$ has two roots. Find these roots. [4]

5 The volume V m^3 of a pile of grain of height h metres is modelled by the equation

$$V = 4\sqrt{h^3 + 1} - 4.$$

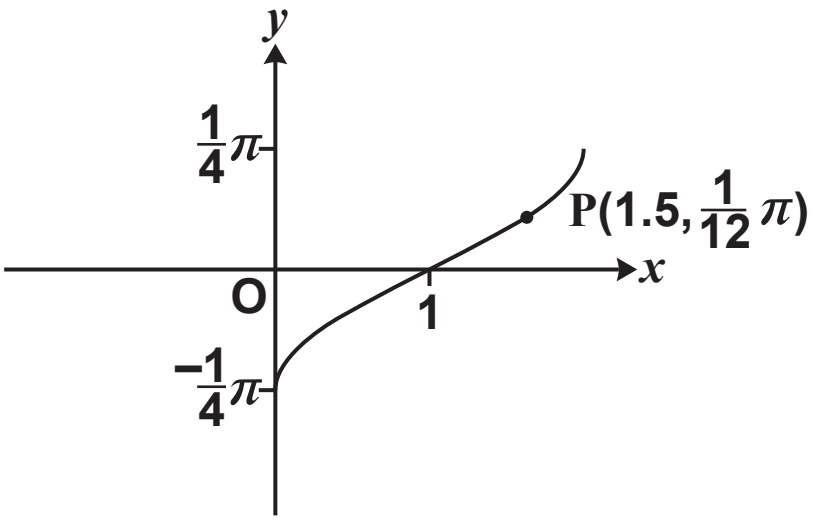
(i) Find $\frac{\mathrm{d}V}{\mathrm{d}h}$ when $h = 2$. [4]

At a certain time, the height of the pile is 2 metres, and grain is being added so that the volume is increasing at a rate of 0.4 m^3 per minute.

(ii) Find the rate at which the height is increasing at this time. [3]

6 Fig. 6 shows part of the curve $\sin 2y = x - 1$. P is the point with coordinates $(1.5, \frac{1}{12}\pi)$ on the curve.

FIG. 6



(i) Find $\frac{dy}{dx}$ in terms of y.

Hence find the exact gradient of the curve $\sin 2y = x - 1$ at the point P. [4]

The part of the curve shown is the image of the curve $y = \arcsin x$ under a sequence of two geometrical transformations.

(ii) Find y in terms of x for the curve $\sin 2y = x - 1$.

Hence describe fully the sequence of transformations. [4]

7 You are given that n is a positive integer.

By expressing $x^{2n} - 1$ as a product of two factors, prove that $2^{2n} - 1$ is divisible by 3. [4]

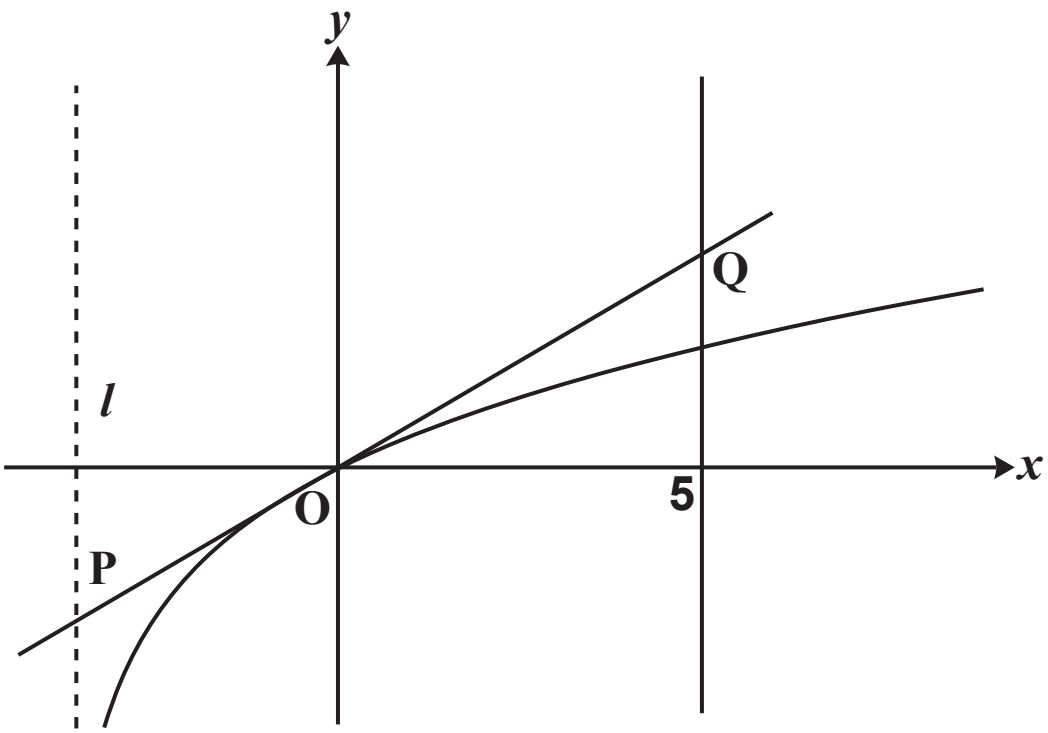
SECTION B (36 marks)

8 Fig. 8 shows the curve $y = \dfrac{x}{\sqrt{x+4}}$ and the line $x = 5$.

The curve has an asymptote l.

The tangent to the curve at the origin O crosses the line l at P and the line $x = 5$ at Q.

FIG. 8



(i) Show that for this curve $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x+8}{2(x+4)^{\frac{3}{2}}}$. [5]

(ii) Find the coordinates of the point P. [4]

(iii) Using integration by substitution, find the exact area of the region enclosed by the curve, the tangent OQ and the line $x = 5$. [9]

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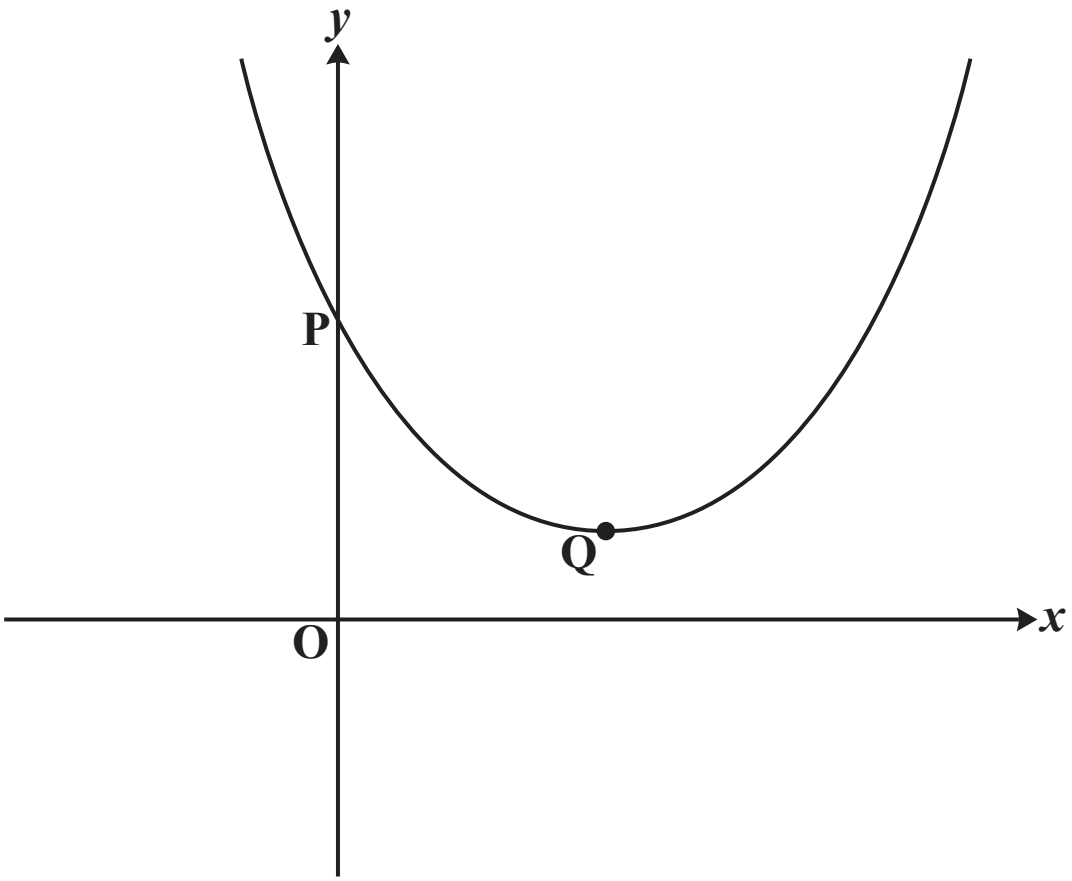
9 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \mathrm{e}^{2x} + k\,\mathrm{e}^{-2x}$ and k is a constant greater than 1.

The curve crosses the y-axis at P and has a turning point Q.

FIG. 9



(i) Find the y-coordinate of P in terms of k. [1]

(ii) Show that the x-coordinate of Q is $\frac{1}{4}\ln k$, and find the y-coordinate in its simplest form. [5]

(iii) Find, in terms of k, the area of the region enclosed by the curve, the x-axis, the y-axis and the line $x = \frac{1}{2}\ln k$.
Give your answer in the form $ak + b$. [4]

The function $g(x)$ is defined by $g(x) = f(x + \frac{1}{4} \ln k)$.

(iv) (A) Show that $g(x) = \sqrt{k}\,(e^{2x} + e^{-2x})$. [3]

(B) Hence show that $g(x)$ is an even function. [2]

(C) Deduce, with reasons, a geometrical property of the curve $y = f(x)$. [3]

END OF QUESTION PAPER

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