OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4754/01A

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

FRIDAY 24 JUNE 2016 – Morning

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book, or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

None

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator
A model for Question 7

READ INSTRUCTIONS OVERLEAF

DC (ST/CGW) 134924/1

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INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

If you use the Printed Answer Book WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

Any blank pages are indicated.

This paper will be followed by
PAPER B: COMPREHENSION.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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SECTION A (36 marks)

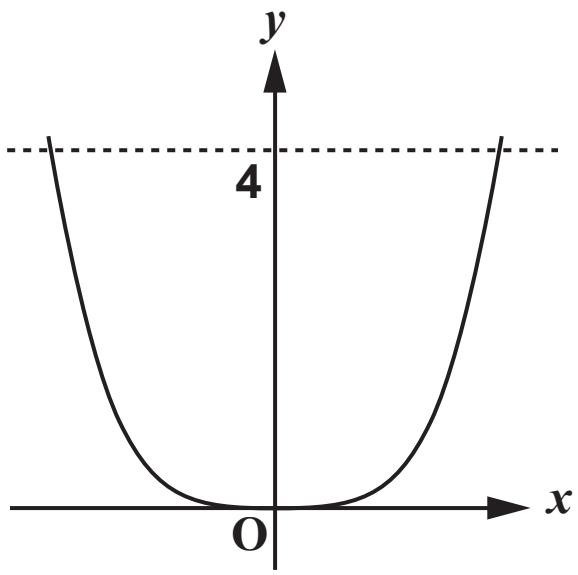
1 Express $\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Hence show that the equation $\cos\theta - 3\sin\theta = 4$ has no solution. [6]

2 Given that $\left(1 + \frac{x}{p}\right)^q = 1 - x + \frac{3}{4}x^2 + \ldots$, find p and q, and state the set of values of x for which the expansion is valid. [7]

3 Fig. 3 below shows the curve $y = x^4$ and the line $y = 4$.

Fig. 3



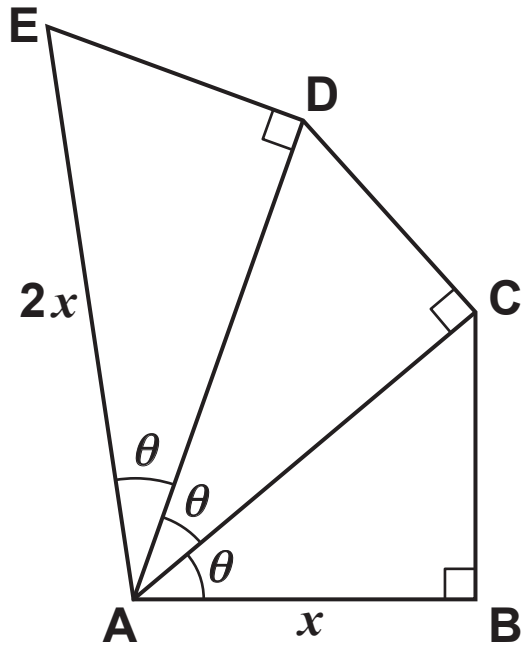
The finite region enclosed by the curve and the line is rotated through 180° about the y-axis. Find the exact volume of revolution generated. [4]

4 Solve the equation $2\sin 2\theta = 1 + \cos 2\theta$ for $0° \leqslant \theta \leqslant 180°$. [5]

5 In Fig. 5 below, triangles ABC, ACD and ADE are all right-angled, and angles BAC, CAD and DAE are all θ.

$AB = x$ and $AE = 2x$.

Fig. 5

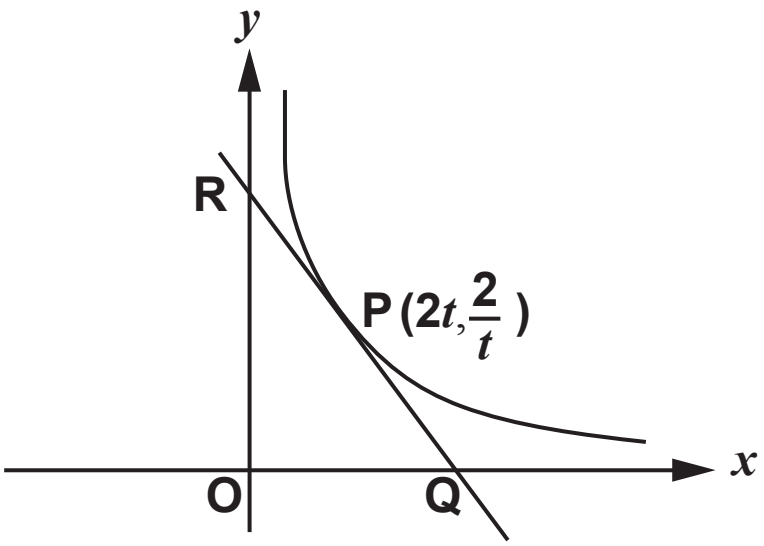


(i) Show that $\sec^3\theta = 2$. [3]

(ii) Hence show the ratio of lengths ED to CB is $2^{\frac{2}{3}} : 1$. [4]

6 P is a general point on the curve with parametric equations $x = 2t$, $y = \frac{2}{t}$. This is shown in Fig. 6 below. The tangent at P intersects the x- and y-axes at the points Q and R respectively.

Fig. 6



Show that the area of the triangle OQR, where O is the origin, is independent of t. [7]

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SECTION B (36 marks)

7 **Fig. 7 opposite shows a 3D cuboid OABCDEFG. The coordinates are as follows:**

Point A (4,0,0)
Point B (4,3,0)
Point C (0,3,0)
Point D (0,0,5)
Point E (4,0,5)
Point F (4,3,5)
Point G (0,3,5)

The point P has coordinates (4,2,0).
A model has also been provided to help you.

(i) **Find the length of the diagonal AG. [2]**

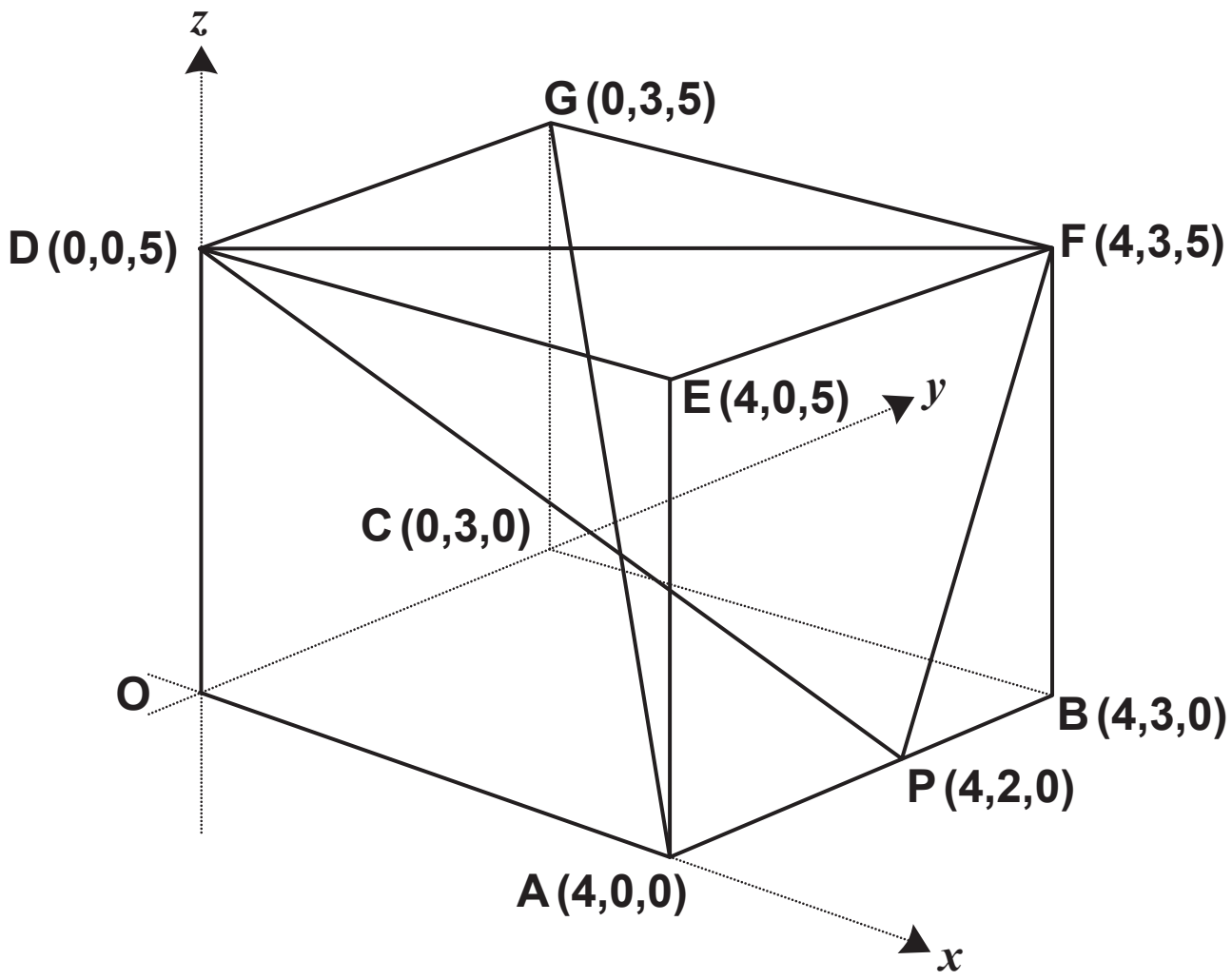
(ii) **Show that the vector $\mathbf{n} = 15\mathbf{i} - 20\mathbf{j} + 4\mathbf{k}$ is normal to the plane DPF. Hence find the cartesian equation of this plane. [6]**

The diagonal AG intersects the plane DPF at Q.

(iii) **Write down a vector equation of the line AG. Hence find the coordinates of the point Q, and the ratio AQ : QG. [6]**

(iv) **Find the acute angle between the line AG and the plane DPF. [4]**

Fig. 7



8 (i) Show that $\frac{1}{2+x}+\frac{1}{2-x}=\frac{4}{(2+x)(2-x)}$. [1]

In a chemical reaction, the time t minutes taken for a mass x mg of a substance to be produced is modelled by the equation

$$t=\ln\left(\frac{2+x}{2-x}\right).$$

(ii) Show that when $t=0$, $x=0$. [2]

(iii) Show that the rate of change of x is proportional to the product of $(2+x)$ and $(2-x)$, and find the constant of proportionality. [4]

(iv) Show that $x=\frac{2\left(1-\mathrm{e}^{-t}\right)}{1+\mathrm{e}^{-t}}$.

Hence determine the long-term mass of the substance predicted by this model. [4]

In another chemical reaction, the mass x mg at time t minutes is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t}=k(2+x)(2-x)\mathrm{e}^{-t},$$

where k is a positive constant, and $x=0$ when $t=0$.

(v) Show by integration that, for this reaction,
$\ln\left(\frac{2+x}{2-x}\right) = 4k\left(1 - \mathrm{e}^{-t}\right)$. [5]

(vi) Given that the long-term mass of this substance is 1.85 mg, find the value of k. [2]

END OF QUESTION PAPER