OXFORD CAMBRIDGE AND RSA EXAMINATIONS

AS GCE

4755/01

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

FRIDAY 20 MAY 2016: Morning

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED 24pt

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:
None

OTHER MATERIALS REQUIRED:
Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (LK/FD) 131840/2

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INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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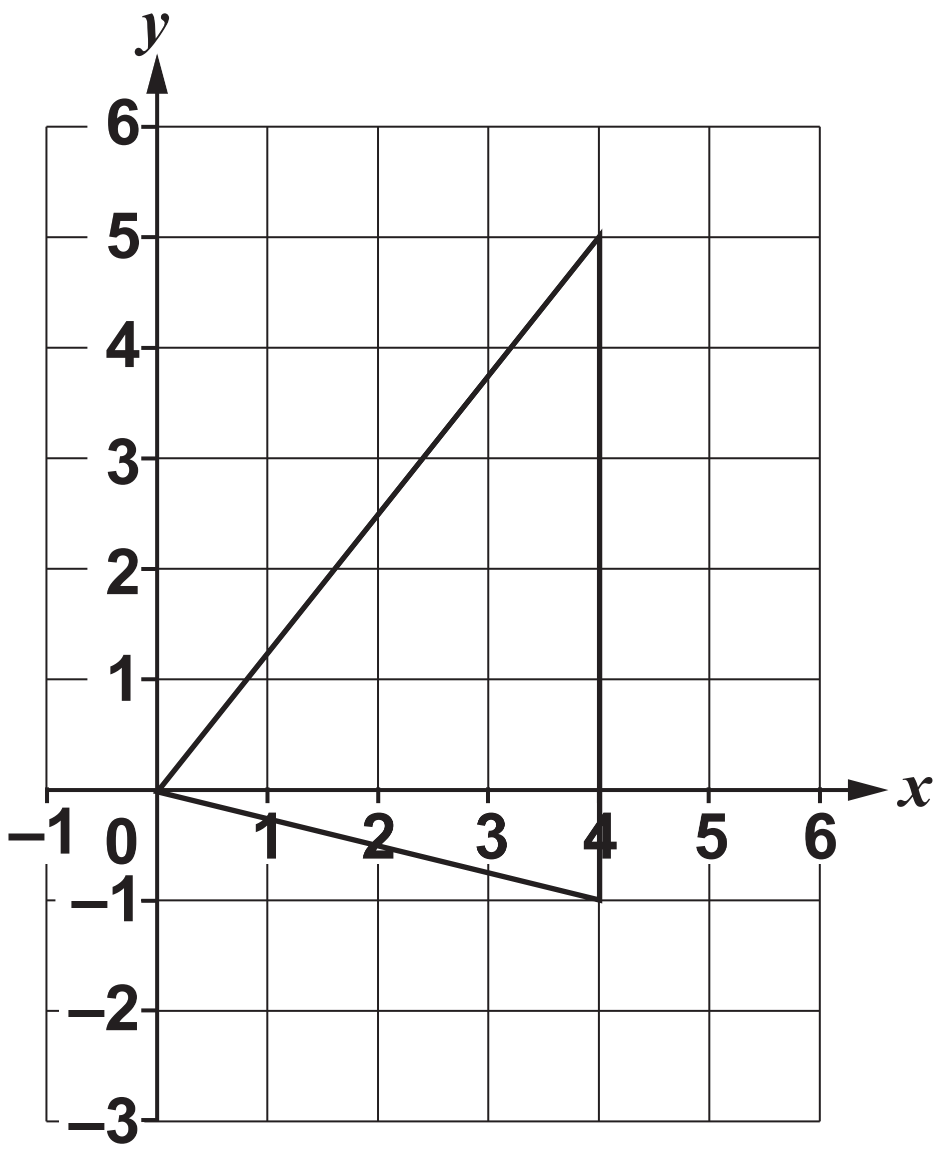
SECTION A (36 marks)

1 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 8 & -2 \\ p & 1 \end{pmatrix}$, where $p \neq -4$.

(i) Find the inverse of $\mathbf{M}$ in terms of p. [2]

(ii)

Fig. 1



The triangle shown in Fig. 1 undergoes the transformation represented by the matrix $\begin{pmatrix} 8 & -2 \\ 3 & 1 \end{pmatrix}$. Find the area of the image of the triangle following this transformation. [2]

2 The complex number z_1 is $2-5\mathrm{j}$ and the complex number z_2 is $(a-1)+(2-b)\mathrm{j}$, where a and b are real.

(i) Express $\dfrac{z_1^*}{z_1}$ in the form $x+y\mathrm{j}$, giving x and y in exact form. You must show clearly how you obtain your answer. [4]

(ii) Given that $\dfrac{z_1^*}{z_1}=z_2$, find the exact values of a and b. [2]

3 You are given that $\mathbf{A}=\begin{pmatrix}\lambda & 6 & -4\\ 2 & 5 & -1\\ -1 & 4 & 3\end{pmatrix}$, $\mathbf{B}=\begin{pmatrix}-19 & 34 & -14\\ 5 & -5 & 5\\ -13 & 18 & -3\end{pmatrix}$

and $\mathbf{AB}=\mu\mathbf{I}$, where $\mathbf{I}$ is the 3×3 identity matrix.

(i) Find the values of λ and μ. [4]

(ii) Hence find $\mathbf{B}^{-1}$. [2]

4 (i) Use standard series to show that

$$\sum_{r=1}^{n} r^2(2r-p)=\frac{1}{6}n(n+1)(3n^2+(3-2p)n-p),$$

where p is a constant. [4]

(ii) Given that the coefficients of n^3 and n^4 in the expression for $\sum\limits_{r=1}^{n} r^2(2r-p)$ are equal, find the value of p. [2]

5 The loci C_1 and C_2 are given by $|z + 3 - 4j| = 5$ and

$\arg(z + 3 - 6j) = \frac{1}{2}\pi$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [5]

(ii) Write down the complex number represented by the point of intersection of C_1 and C_2. [1]

(iii) Indicate, by shading on your sketch, the region satisfying

$|z + 3 - 4j| \geqslant 5$ and $\frac{1}{2}\pi \leqslant \arg(z + 3 - 6j) \leqslant \frac{3}{4}\pi$. [2]

6 A sequence is defined by $u_1 = 8$ and $u_{n+1} = 3u_n + 2n + 5$.

Prove by induction that $u_n = 4(3^n) - n - 3$. [6]

SECTION B (36 marks)

7 The function $f(z) = 2z^4 - 9z^3 + Az^2 + Bz - 26$ has real coefficients. The equation $f(z) = 0$ has two real roots, α and β, where $\alpha > \beta$, and two complex roots, γ and δ, where $\gamma = 3 + 2\text{j}$.

(i) Show that $\alpha + \beta = -\frac{3}{2}$ and find the value of $\alpha\beta$. [5]

(ii) Hence find the two real roots α and β. [3]

(iii) Find the values of A and B. [3]

(iv) Write down the roots of the equation $f\left(\frac{w}{\text{j}}\right) = 0$. [2]

8 A curve has equation $y = \dfrac{3x^2 - 9}{x^2 + 3x - 4}$.

(i) Find the equations of the two vertical asymptotes and the one horizontal asymptote of this curve. [3]

(ii) State, with justification, how the curve approaches the horizontal asymptote for large positive and large negative values of x. [3]

(iii) Sketch the curve. [3]

(iv) Solve the inequality $\dfrac{3x^2 - 9}{x^2 + 3x - 4} \geqslant 0$. [3]

9 You are given that

$$\frac{3}{4(2r-1)} - \frac{1}{2r+1} + \frac{1}{4(2r+3)} = \frac{2r+5}{(2r-1)(2r+1)(2r+3)}.$$

(i) Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{2r+5}{(2r-1)(2r+1)(2r+3)} = \frac{2}{3} - \frac{3}{4(2n+1)} + \frac{1}{4(2n+3)}. \quad [6]$$

(ii) Write down the limit to which $\sum_{r=1}^{n} \frac{2r+5}{(2r-1)(2r+1)(2r+3)}$ converges as n tends to infinity. [1]

(iii) Find the sum of the finite series

$$\frac{45}{39 \times 41 \times 43} + \frac{47}{41 \times 43 \times 45} + \frac{49}{43 \times 45 \times 47} + \ldots + \frac{105}{99 \times 101 \times 103},$$

giving your answer to 3 significant figures. [4]

END OF QUESTION PAPER

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