OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4756/01

MATHEMATICS (MEI)

Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

MONDAY 27 JUNE 2016: Morning

DURATION: 1 hour 30 minutes plus your additional time allowance

MODIFIED ENLARGED 24pt

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:
None

OTHER MATERIALS REQUIRED:
Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (LK) 134031/1

7141931063

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

SECTION A (54 marks)

1 (a) (i) Given that $f(x) = \arctan x$, write down an expression for $f'(x)$. Assuming that x is small, use a binomial expansion to express $f'(x)$ in ascending powers of x as far as the term in x^4. [3]

(ii) Hence express $\arctan x$ in ascending powers of x as far as the term in x^5. [3]

(b) Find, in exact form, the value of the following integral.

$$\int_0^{\frac{3}{4}} \frac{1}{\sqrt{3-4x^2}}\,dx$$ [5]

(c) A curve has polar equation $r = \dfrac{a}{\sqrt{\theta}}$ where $a > 0$.

(i) Sketch the curve for $\frac{\pi}{4} \leqslant \theta \leqslant 2\pi$. [2]

(ii) State what happens to r as θ tends to zero. [1]

(iii) Find the area of the region enclosed by the part of the curve sketched in part (i) and the lines $\theta = \frac{\pi}{4}$ and $\theta = 2\pi$. Give your answer in an exact simplified form. [4]

2 (a) (i) Express $2\sin\frac{1}{2}\theta\left(\sin\frac{1}{2}\theta - \mathrm{j}\cos\frac{1}{2}\theta\right)$

in terms of z where $z = \cos\theta + \mathrm{j}\sin\theta$. [3]

(ii) The series C and S are defined as follows.

$$C = 1 - \binom{n}{1}\cos\theta + \binom{n}{2}\cos 2\theta - \ldots + (-1)^n\binom{n}{n}\cos n\theta$$

$$S = -\binom{n}{1}\sin\theta + \binom{n}{2}\sin 2\theta - \ldots + (-1)^n\binom{n}{n}\sin n\theta$$

Show that

$$C + \mathrm{j}S = \left\{-2\mathrm{j}\sin\tfrac{1}{2}\theta\left(\cos\tfrac{1}{2}\theta + \mathrm{j}\sin\tfrac{1}{2}\theta\right)\right\}^n.$$

Hence show that, for even values of n,

$\dfrac{C}{S} = \cot\left(\frac{1}{2}n\theta\right)$. [8]

(b) Write the complex number $z = \sqrt{6} + \mathrm{j}\sqrt{2}$ in the form $r\mathrm{e}^{\mathrm{j}\theta}$, expressing r and θ as simply as possible.

Hence find the cube roots of z in the form $r\mathrm{e}^{\mathrm{j}\theta}$.

Show the points representing z and its cube roots on an Argand diagram. [7]

3 (i) Find the eigenvalues and eigenvectors of the matrix $\mathbf{M}$, where

$$\mathbf{M} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}.$$

Hence express $\mathbf{M}$ in the form $\mathbf{PDP}^{-1}$ where $\mathbf{D}$ is a diagonal matrix. [8]

(ii) Write down an equation for $\mathbf{M}^n$ in terms of the matrices $\mathbf{P}$ and $\mathbf{D}$.

Hence obtain expressions for the elements of $\mathbf{M}^n$.

Show that $\mathbf{M}^n$ tends to a limit as n tends to infinity. Find that limit. [6]

(iii) Express $\mathbf{M}^{-1}$ in terms of the matrices $\mathbf{P}$ and $\mathbf{D}$. Hence determine whether or not $(\mathbf{M}^{-1})^n$ tends to a limit as n tends to infinity. [4]

SECTION B (18 marks)

4 (i) Given that $y = \cosh x$, use the definition of $\cosh x$ in terms of exponential functions to prove that

$$x = \pm \ln\left(y + \sqrt{y^2 - 1}\right).$$ [5]

(ii) Solve the equation

$$\cosh x + \cosh 2x = 5,$$

giving the roots in an exact logarithmic form. [5]

(iii) Sketch the curve with equation $y = \cosh x + \cosh 2x$. Show on your sketch the line $y = 5$.

Find the area of the finite region bounded by the curve and the line $y = 5$. Give your answer in an exact form that does not involve hyperbolic functions. [8]

END OF QUESTION PAPER