OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4758/01

MATHEMATICS (MEI)

Differential Equations

QUESTION PAPER

FRIDAY 17 JUNE 2016: Afternoon

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

None

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (RW) 131834/1

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INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer any THREE questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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1 The differential equation

$$4\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 8\frac{\mathrm{d}x}{\mathrm{d}t} + 3x = \mathrm{f}(t)$$

is to be solved.

Consider first the case where $\mathrm{f}(t) = 4\mathrm{e}^{0.5t}$.

(i) Find the general solution for x. [7]

(ii) Given that when $t = 0$, $x = 6$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = -4$, find the particular solution for x. [4]

(iii) Given that x has a minimum value, find the value of t for which this minimum occurs. [4]

Now consider the case where $\mathrm{f}(t) = 4\mathrm{e}^{-0.5t}$.

(iv) Given that when $t = 0$, $x = 6$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = -4$, find the particular solution for x. [9]

2 (a) The differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} - 3y = x^5 \cos x$$

is to be solved.

(i) Find the general solution for y in terms of x. [8]

(ii) Find the particular solution for which $y = 0$ when $x = \frac{1}{2}\pi$. [2]

(b) Now consider the differential equation

$$\sec x\frac{\mathrm{d}y}{\mathrm{d}x} - 3y^2 = 0.$$

(i) Find the particular solution for which $y = 1$ when $x = \frac{1}{2}\pi$. [7]

(ii) Show that the maximum value of y is 1. [2]

(c) Now consider the differential equation

$$\sec x\frac{\mathrm{d}y}{\mathrm{d}x} - 3y^2 = x,$$ **where $y = 0$ when $x = 1$.**

This is to be solved numerically using Euler's method. The algorithm is given by

$$x_{r+1} = x_r + h, \qquad y_{r+1} = y_r + hy'_r.$$

Use a step length of 0.01 to estimate y when $x = 1.02$. [5]

3 A parachutist of mass 90 kg falls vertically from a stationary helicopter. When he is x m below the helicopter, his velocity is v m s^{-1}. The forces acting on the parachutist are his weight and a resistive force of magnitude $0.36v^2$ N.

(i) Show that his motion can be modelled by the differential equation

$$v\frac{\mathrm{d}v}{\mathrm{d}x} = 9.8 - 0.004v^2.$$ [2]

(ii) Solve this differential equation to show that

$$v^2 = 2450(1 - \mathrm{e}^{-0.008x}).$$ [6]

(iii) Sketch the graph of v against x. [2]

(iv) Find how far the parachutist has fallen when his speed has reached 48 m s^{-1}. [2]

The parachutist opens his parachute when his speed is 48 m s^{-1}. The forces acting on him now are his weight and a resistive force of magnitude $72v$ N.

(v) Find an expression for v in terms of t, where t is the time in seconds that has elapsed since the parachute was opened. [8]

(vi) Find the distance that the parachutist falls during the first 5 seconds after his parachute has opened. [4]

4 The simultaneous differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x - y + 3\cos t$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 5x - y - 12\sin t$$

are to be solved for $t \geqslant 0$, where t denotes time.

(i) Eliminate y to obtain a second order differential equation for x in terms of t. Hence find the general solution for x. [12]

(ii) Find the corresponding general solution for y. [3]

When $t = 0$, $y = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 5$.

(iii) Find the particular solutions for x and y. [4]

(iv) Find the time that elapses between the first two occasions on which $x = y$. [5]

END OF QUESTION PAPER