OCR
Oxford Cambridge and RSA

# GCE

# Mathematics (MEI)

Unit **4752**: Concepts for Advanced Mathematics

Advanced Subsidiary GCE

# Mark Scheme for June 2016

**Annotations and abbreviations**

| Annotation in scoris | Meaning |
|---|---|
| ✔ and �’ | |
| **BOD** | Benefit of doubt |
| **FT** | Follow through |
| **ISW** | Ignore subsequent working |
| **M0** , **M1** | Method mark awarded 0, 1 |
| **A0** , **A1** | Accuracy mark awarded 0, 1 |
| **B0** , **B1** | Independent mark awarded 0, 1 |
| **SC** | Special case |
| **∧** | Omission sign |
| **MR** | Misread |
| Highlighting | |
| | |
| Other abbreviations in mark scheme | Meaning |
| E1 | Mark for explaining |
| U1 | Mark for correct units |
| G1 | Mark for a correct feature on a graph |
| M1 dep* | Method mark dependent on a previous mark, indicated by * |
| cao | Correct answer only |
| oe | Or equivalent |
| rot | Rounded or truncated |
| soi | Seen or implied |
| www | Without wrong working |

**Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand**

a      Annotations should be used whenever appropriate during your marking.

       **The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks.** It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

       For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b      An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

       Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c      The following types of marks are available.

       **M**
       A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

       **A**
       Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

       **B**
       Mark for a correct result or statement independent of Method marks.

       **E**
       A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d       When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e       The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f       Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g       Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h     For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units.  This is achieved by withholding one A mark in the question.

       Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| 1 | (i) | | $kx^{\frac{1}{2}-1}$ or $kx^{-\frac{1}{2}}$ seen | **M1** | $k > 0$ | **B2** for correct answer unsupported |
| | | | $3x^{-\frac{1}{2}}$ or $\dfrac{3}{\sqrt{x}}$ isw | **A1** | **A0** for eg $3x^{-\frac{1}{2}} + c$ | |
| | | | | **[2]** | | |
| 1 | (ii) | | $kx^{-2+1}$ or $kx^{-1}$ oe seen | **M1** | for any non-zero $k$ | **SC0** for $\dfrac{12}{2x}$ or $\dfrac{6}{x}$ |
| | | | $-12x^{-1}$ or $-\dfrac{12}{x}$ or $\dfrac{-12}{x}$ isw | **A1** | | **A0** for $\dfrac{12}{-x}$ or $\dfrac{12x^{-1}}{-1}$ |
| | | | $+ c$ | **A1** | seen at least once following integration | do not allow MR for integration of $12x^2$ |
| | | | | **[3]** | | |
| 2 | (i) | | (i) [5], 10, 5, [10] | **M1** | ignore extra terms | condone wrongly attributed terms |
| | | | [10 + 5 + 10 =] 25 | **A1** | not from wrong working | **B2** for 25 unsupported |
| | | | (ii) 0 | **B1** | | |
| | | | | **[3]** | | |
| 3 | | | $1.5 + (4-1)d = 12$ or better | **M1** | or $1.5 \times r^{(4-1)} = 12$ or better | if first **M0 B0** allow **B3** for $d = 3.5$ **and** $r = 2$; |
| | | | $d = 3.5$ | **A1** | $r = 2$ | **B2** for one of these; may be embedded in calculation of difference |
| | | | $r = 2$ | **B1** | $d = 3.5$ | |
| | | | $1.5 \times$ their $2^9 - (1.5 + 9 \times$ their $3.5)$ oe | **M1** | **M0** for use of their $S_{10}$ in either term | **NB** $768 - 33$ |
| | | | difference $= 735$ | **A1** | | allow $-735$ |
| | | | | **[5]** | | |

| Question | Answer | Marks | Guidance | |
|---|---|---|---|---|
| 4 | $5.6^2 + 7.2^2 - 2 \times 5.6 \times 7.2 \times \cos 68$ seen | **M1** | may be implied by 53 or BC in range | |
| | 53 or 53.0 | **M1** | may be implied by BC in range | **NB** 52.9917243; (allow 47.7 to 47.71 from calculator in radian mode; may be implied by 6.90 to 6.91) |
| | [BC =] 7.3 or 7.27 to 7.28 | **A1** | **NB** 7.27954… | |
| | $\sin C = \dfrac{7.2 \times \sin 68}{\text{their } BC}$ | **M1** | or $[\cos C] = \dfrac{\text{their } BC^2 + 5.6^2 - 7.2^2}{2 \times 5.6 \times \text{their } BC}$ | |
| | 66 or awrt 66.5 | **A1** **[5]** | allow 1.2 or awrt 1.16 (radians); **A0** for eg 1.2 degrees | **NB** $\sin C = 0.917053\ldots$ $\cos C = 0.398766\ldots$ |
| | *Alternatively* eg if the perpendicular from B to AC, BX, is used | | | eg if perpendicular from C to AB, CY, is used, mark as follows |
| | $7.2 \times \cos 68$ seen | **M1\*** | if unsupported, **B2** for 2.70 or better | $5.6 \times \cos 68$ seen |
| | 2.7 or 2.697 to 2.70 | **A1** | | 2.1 or 2.097 to 2.10 |
| | XC = 5.6 – their AX | **M1dep\*** | **NB** 2.902832527 | BY = 7.2 – their AY |
| | $\tan C = \left[\dfrac{\text{BX}}{\text{XC}}\right] = \dfrac{7.2 \times \sin 68}{\text{their XC}}$ | **M1** | | $\tan B = \left[\dfrac{\text{CY}}{\text{BY}}\right] = \dfrac{5.6 \times \sin 68}{\text{their BY}}$ |
| | 66 or awrt 66.5 | **A1** **[5]** | allow 1.2 or awrt 1.19 (radians); **A0** for eg 1.2 degrees | C [= 90 – B] = 66 or awrt 66.5 |

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| 5 | (i) | | $\sin kx$ | M1 | $k > 0$ and $k \neq 1$ | condone use of other variable |
| | | | $y = \sin 2x$ | A1 | must see "$y =$" at some stage for A1 | condone $f(x) = \sin 2x$ |
| | | | | [2] | | |
| 5 | (ii) | | sketch of sine curve with period 360° and amplitude 1 | B1 | for $0 \leq x \leq 450$; ignore curve outside this range; <br><br> do not allow sketch of $y = \cos x$ or $y = -\sin x$ for either mark | amplitude, period and centring on $y = -3$ must be clear from correct numerical scale, numerical labelling or comment; strokes on axes insufficient to imply scale: mark intent |
| | | | sine curve centred on $y = -3$ and starting at $(0, -3)$ | B1 | | allow full marks if $y = \sin x$ and $y = \sin x - 3$ seen on same diagram |
| | | | | [2] | | |
| 6 | (i) | | $\dfrac{1}{2} r^2 \theta$ or $\dfrac{1}{2} a^2 \sin\theta$ or $a^2 \sin\dfrac{1}{2}\theta \cos\dfrac{1}{2}\theta$ seen | M1 | do not allow use of variable other than $\theta$ | allow eg <br> $\dfrac{\theta}{2\pi} \times \pi r^2$ or $\dfrac{1}{2} a^2 \sin\left(\dfrac{180\theta}{\pi}\right)$ seen |
| | | | $\dfrac{1}{2} r^2 \theta - \dfrac{1}{2} a^2 \sin\theta$ isw oe | A1 | | oe |
| | | | | [2] | | |
| 6 | (ii) | | $\dfrac{1}{2} a^2 \sin 0.8 = \dfrac{1}{2} \times 12^2 \times 0.8 - \dfrac{1}{2} a^2 \sin 0.8$ oe | B1 | or eg $\dfrac{1}{2} a^2 \sin 0.8 = \dfrac{1}{4} \times 12^2 \times 0.8 \ [= 28.8]$ or equivalent in degrees NB $\theta = 45.8366236\ldots°$ | NB $a^2 = \dfrac{57.6}{0.717356} = 80.29485$ <br><br> NB $\theta = 45.83662361\ldots°$ |
| | | | [$a =$] 8.96 cao; mark the final answer | B1 | if unsupported, allow B2 for 8.96 or allow B1 for 9.0 or 8.96074…to 4 sf or more | NB $\dfrac{1}{2} \sin 0.8 = 0.35867..$ |
| | | | | [2] | | |

| Question | | Answer | Marks | Guidance |
|---|---|---|---|---|
| 7 | (i) | substitution of <br> $\tan x = \dfrac{\sin x}{\cos x}$ or $\sqrt{1-\sin^2 x} = \sqrt{\cos^2 x}$ or $\cos x$ <br> in given LHS <br><br> both substitutions seen and completion to $\sin x$ as final answer | **M1** <br><br><br><br> **A1** <br><br><br> **[2]** | if no substitution, statements must follow a logical order and the argument must be clear; if one substitution made correctly, condone error in other part of LHS <br><br> **NB AG**; answer must be stated <br><br> allow consistent use of other variable eg $\theta$ for both marks | condone omission of variable throughout for **M1** only **,** but allow recovery from omission of variable at end <br><br> **M0** if first move is to square one or both sides <br><br> Simply stating eg $\tan x = \dfrac{\sin x}{\cos x}$ is insufficient <br><br> *Alternatively* **SC2** for complete argument eg <br> $\tan x = \dfrac{\sin x}{\cos x}$ <br> $[\tan x \times \cos x = \sin x]$ <br> $\sin^2 x + \cos^2 x = 1$ <br> $\cos x = \sqrt{1 - \sin^2 x}$ <br> $\tan x = \dfrac{\sin x}{\sqrt{1 - \sin^2 x}}$ <br> $\tan x \times \sqrt{1 - \sin^2 x} = \sin x$ oe |
| 7 | (ii) | 0, 180, 360 <br><br> 14 or 14.47 to 14.5 <br><br> 166 or awrt 165.5 | **B1** <br><br> **B1** <br><br> **B1** <br><br><br> **[3]** | all 3 required <br><br> radians: mark as scheme but deduct one from total <br> 0, π, 2π; <br> 0.25 or 0.253 or awrt 0.2527; <br> 2.89 or 2.889 or awrt 2.8889 | **NB** sin $y$ = 0 or ¼ <br><br> ignore extra values outside range <br><br> if **B3,** deduct 1 mark for extra values within range |

| | Question | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| 8 | (i) | | $\log_a 1 = 0$ soi or $3m\log_a a$ or $\log_a a^{-3m}$ seen | M1 | do not condone $3m\log a$ | do not allow MR for $(\log_a a^m)^3$ |
| | | | $-3m$ cao | A1 [2] | | |
| 8 | (ii) | | $(2x+1)\log_3 3 = \log_3 1000$ or $2x+1 = \log_3 1000$ oe | M1 | Or $(2x+1)\log_{10} 3 = \log_{10} 1000$ $[= 3]$ | condone omission of brackets; allow omission of base 10 or consistent use of other base |
| | | | $[x=]\dfrac{\log_3 1000 - 1}{2}$ oe | M1 | or $[x=]\dfrac{\dfrac{3}{\log_{10} 3} - 1}{2}$ oe | allow one sign error and / or omission of brackets |
| | | | | | | allow recovery from bracket error for **A1** |
| | | | 2.64 cao; mark the final answer | A1 [3] | not from wrong working | 0 if unsupported or for answer obtained by trial and error on $3^{2x+1} = 1000$ |
| 9 | (i) | | $\dfrac{h}{2} \times (0 + 0 + 2[4 + 4.9 + 5 + 4.9 + 4])$ oe | M1 | correct formula used with 4, 5 or 6 strips and numerical value for *h*; condone omission of zeros or omission of outer brackets for both **M** marks | allow eg $\frac{1}{2}\times 1 \times (4 + 4 + 2[4.9 + 5 + 4.9])$ $\frac{1}{2}\times 1 \times (4 + 0 + 2[4 + 4.9 + 5 + 4.9])$ (**NB** may be implied by 18.8 & 20.8 respectively) |
| | | | all non-zero *y*-values correctly placed | M1 | **M0M0** if 1, 2, 3 or 6 used as *y*-values (these are *x*-values) | |
| | | | $h = 1$ used in formula or consistently with two triangles and four trapezia | B1 | if **M0M0** allow **B1** for $h = 1$ and **B2** for 22.8 from area of 4 trapezia and 2 triangles and **B1** for 1140 | |
| | | | area = 22.8 **and** volume = 1140 isw cao | A1 [4] | ignore units | if **M0M0B0** allow **SC4** for 22.8 and 1140 obtained correctly by other method |

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| 9 | (ii) | A | substitution of $x = 1.2$ or $4.8$ to find $y$ | M1 | allow substitution of $1.2 \le x \le 1.234$ or $4.766 \le x \le 4.8$ | or **M1** for $y = 4.4$, $x = 1.234$ [or 4.766] and |
| | | | $y = 4.35$ or $4.352$ and correct comparison with 4.4 isw | A1 | | **A1** for comparison of 1.234 with 1.2 or 4.766 with 4.8 [so gap less than 3.6] |
| | | | | **[2]** | | |
| 9 | (ii) | B | $F[x] = \dfrac{5}{81}\left( \dfrac{108}{2}x^2 - \dfrac{54}{3}x^3 + \dfrac{12}{4}x^4 - \dfrac{x^5}{5} \right)$ oe | M2 | **M1** for 3 correct terms; ignore $+c$ | condone omission of $\dfrac{5}{81}$; |
| | | | eg $\dfrac{10}{3}x^2 - \dfrac{10}{9}x^3 + \dfrac{5}{27}x^4 - \dfrac{1}{81}x^5$ | | allow coefficients 3.333333…, 1.11111…, 0.185185.., 0.01234567…r.o.t to 2 sf or better<br><br>or decimal equivalents in numerator: 6.6666…, 3.333333…, 0.74074…, 0.061728… r.o.t to 2 sf or better | **M0** if $\dfrac{5}{81}x$ seen outside bracket but next **M1** is still available; ignore subsequent attempt to evaluate $c$ for first **M2** |
| | | | $F[6] - F[0]$ or $2 \times (F[3] - F[0])$ | M1 | dependent on at least two terms correctly integrated in bracket; condone omission of $-F(0)$ | **M0** for non-zero lower limit |
| | | | 24 | A1 | | 24 unsupported does not score |
| | | | 1200 | B1 | | ignore units |
| | | | | **[5]** | | |
| 10 | (i) | | $\dfrac{\left(5.1^2 - 10.2\right) - \left(5^2 - 10\right)}{5.1 - 5}$ oe | M1 | condone omission of brackets | 0 for 8.1 unsupported |
| | | | 8.1 | A1 | | |
| | | | | **[2]** | | |

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| **10** | **(ii)** | | $\dfrac{(5+h)^2 - 2(5+h) - \text{their } 15}{h}$ oe | **M1** | condone omission of brackets | |
| | | | $25 + 10h + h^2 - 10 - 2h$ oe seen | **M1** | allow one sign error | |
| | | | numerator is $8h + h^2$ | **A1** | | |
| | | | $8 + h$ isw | **A1** **[4]** | | |
| **10** | **(iii)** | | $h \to 0$ | **M1** | may be embedded; allow eg "tends to 0" | **M0** for differentiation of $x^2 - 2x$ <br> **M0** for following from part (i) <br> **M0** for $h = 0$ |
| | | | their 8 | **A1** **[2]** | **FT** their $k + h$ from part (ii) | |
| **10** | **(iv)** | | $y = 8x - 25$ isw | **B1** | or $y - 15 = 8\,(x - 5)$ isw <br> or $y = 8x + c$ and $c = -25$ stated isw | |
| | | | non-zero numerical value for $x$-intercept on their straight line found | **M1** | | |
| | | | $[\,x =\,]$ 3.125 oe | **A1** | may be embedded in calculation for area | |
| | | | $\dfrac{1}{2} \times$ their non-zero $y$-intercept $\times$ their $\dfrac{25}{8}$ | **M1** | condone arithmetic slips in finding values of intercepts | or integration and evaluation of their $\displaystyle\int_{0}^{25/8} (8x - 25)\mathrm{d}x$ ; <br> lower limit must be 0 |
| | | | $\dfrac{625}{16}$ or $39\frac{1}{16}$ or 39.0625 isw | **A1** **[5]** | accept rounded to 1 dp or better for **A1**; but **A0** if **final** answer negative | |

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| **11** | **(i)** | | $\log_{10} y = \log_{10} a + bt$ www | **B1** | **B0** for just $\log_{10} y = \log_{10} a + bt \log_{10} 10$ | allow omission of base throughout question |
| | | | gradient is $b$, intercept is $\log_{10} a$ cao | **B2** | **B1** for one correct; award independently of their equation; must be stated – linking by arrows etc is insufficient; condone $m = b$ and $c = \log a$ | ignore $t$-intercept is $\dfrac{-\log_{10} a}{b}$ **B0** for gradient is $bt$ |
| | | | | **[3]** | | |
| **11** | **(ii)** | | 1.58, 1.8[0], 1.98, 2.37, 2.68 | **B1** | allow values which round to these numbers to 2 dp; | all values must be correct |
| | | | all values correct and all plotted accurately | **B1** | within tolerance on overlay; | |
| | | | ruled line of best fit for at least $1 \le t \le 10$ | **B1** | within tolerance on overlay: must not cut red or green line; line between (1, 0.6) and (1, 1.05) at lower limit and between (10, 2.3) and (10, 2.75) at upper limit; | use ruler tool to check if line is ruled where necessary; tolerance: one small square horizontally at each end; not dependent on correct plots |
| | | | evaluation of $\dfrac{\log y_2 - \log y_1}{t_2 - t_1}$ or substitution of $(t_1, \ \log y_1)$ and $(t_2, \ \log y_2)$ in $\log y = bt + \log a$ to obtain a numerical value for the gradient | **M1** | $(t_1, \ \log y_1)$ and $(t_2, \ \log y_2)$ are points on their line | condone use of values from table |
| | | | | | gradient must be identified as $b$ for **A1** | |
| | | | $0.14 \le b \le 0.24$ | **A1** | | |
| | | | $2.5 \ \le \ a \ \le 6.3$ | **B1** | must be identified as $a$ ; not from wrong working | |
| | | | $y = $ their $a \times 10^{\text{their}b \times t}$ or $y = 10^{\text{their}bt \,+\, \text{their} \log a}$ or $10^{\text{their} \log a} \times 10^{\text{their}b \times t}$ oe | **M1** | | if **M0A0B0M0** allow **SC3** for substitution directly into given formula to obtain $y = a10^{bt}$ with $a$ **and** $b$ in acceptable range |
| | | | $a$ and $b$ or $\log a$ and $b$ both in acceptable range | **A1** | $0.4 \le \log a \ \le \ 0.8$ | |
| | | | | **[8]** | | |
| **11** | **(iii)** | | 260 **or** 261 | **B1** | **B0** for non-integer answer | |
| | | | | **[1]** | | |

**OCR (Oxford Cambridge and RSA Examinations)**
**1 Hills Road**
**Cambridge**
**CB1 2EU**

**OCR Customer Contact Centre**

**Education and Learning**
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Facsimile: 01223 552627
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