



GCE

Mathematics (MEI)

Unit **4753**: Methods for Advanced Mathematics

Advanced GCE

Mark Scheme for June 2016

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.




All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

| Annotation in scoris | Meaning |
|---|--|
|  and  | |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0 M1 , | Method mark awarded 0, 1 |
| A0 A1 , | Accuracy mark awarded 0, 1 |
| B0 B1 , | Independent mark awarded 0, 1 |
| SC | Special case |
|  | Omission sign |
| MR | Misread |
| Highlighting | |
| | |
| Other abbreviations in mark scheme | Meaning |
| E1 | Mark for explaining |
| U1 | Mark for correct units |
| G1 | Mark for a correct feature on a graph |
| M1 dep* | Method mark dependent on a previous mark, indicated by * |
| cao | Correct answer only |
| oe | Or equivalent |
| rot | Rounded or truncated |
| soi | Seen or implied |
| www | Without wrong working |

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
 - g Rules for replaced work
- If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

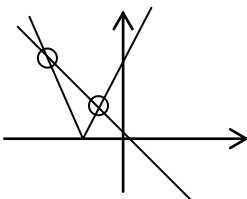
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

- h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

| Question | | | Answer | Marks | Guidance | |
|----------|--|--|--|-------|--|--|
| 1 | | | $\int_0^{\frac{\pi}{2}} (1 + \cos \frac{1}{2}x) dx = \left[x + 2 \sin \frac{1}{2}x \right]_0^{\frac{\pi}{2}}$ | B1 | $\left[x + 2 \sin \frac{1}{2}x \right]$ | |
| | | | $= \frac{\pi}{2} + 2 \sin \frac{\pi}{4} [-0]$ | M1 | substituting limits (upper – lower) | allow 1 slip |
| | | | $= \frac{\pi}{2} + \sqrt{2}$ | A1cao | must be exact, not $2/\sqrt{2}$ | isw from correct answer seen |
| | | | | [3] | | |
| 2 | | | $fg(x) = \ln(2 + e^x)$ | M1 | condone missing brackets | |
| | | | $\Rightarrow \ln(2 + e^x) = 2x$ | A1 | | |
| | | | $\Rightarrow 2 + e^x = e^{2x}$ | | | |
| | | | $\Rightarrow e^{2x} - e^x - 2 = 0$ | M1 | Rearranging into a quadratic in e^x | may be implied from both correct roots |
| | | | $\Rightarrow (e^x - 2)(e^x + 1) = 0, e^x = 2, -1$ | A1 | obtaining roots 2, -1 | -1 root may be inferred from factorising |
| 3 | | | $\Rightarrow e^x = 2, x = \ln 2$ | A1 | $x = \ln 2$ only, not from ww | $x = \ln(-1)$ is A0 |
| | | | | [5] | | |
| | | | let $u = \ln x, u' = 1/x, v' = x^{-1/2}, v = kx^{1/2}$ | M1 | soi ($k \neq 0$) | |
| | | | $\int x^{-1/2} \ln x [dx] = \left[2x^{1/2} \ln x \right] - \int 2x^{1/2} \cdot \frac{1}{x} [dx]$ | A1 | | |
| | | | $= \left[2x^{1/2} \ln x \right] - \int 2x^{-1/2} [dx]$ | M1 | $x^{1/2} / x = x^{-1/2}$ or $1/x^{1/2}$ seen | |
| | | | $= \left[2x^{1/2} \ln x - 4x^{1/2} \right]_1^4$ | A1 | $2x^{1/2} \ln x - 4x^{1/2}$ | may be integrated separately |
| | | | $= 4 \ln 4 - 8 - (2 \ln 1 - 4)$ | | | |
| | | | $= 4 \ln 4 - 4$ | A1cao | oe (eg $\ln 256 - 4$) but must evaluate $\ln 1 = 0$ | mark final answer |
| | | | | [5] | | |

| Question | | | Answer | Marks | Guidance | |
|----------|------|--|---|-------------------------------------|--|---|
| 4 | | |  <p> $x = -1$ $x = -1/3$ </p> | M1 A1 B1 B1 [4] | Sketch of $y = 2x + 1 $ $y = -x$ and two intersections indicated not from ww, condone $(-1, 1)$ not from ww, condone $(-1/3, 1/3)$ | condone no intercept labels, but must be a 'V' shape with vertex on $-ve$ x axis squaring: $(2x+1)^2 = x^2 \Rightarrow 3x^2 + 4x + 1 = 0$ $\Rightarrow (3x+1)(x+1) = 0, x = -1, -1/3$ |
| 5 | (i) | | $dV/dh = 4.1/2(h^3 + 1)^{-1/2} .3h^2$ when $h = 2, dV/dh = 8$ | M1 A1 M1 A1cao [4] | chain rule correct substituting $h = 2$ into their derivative | their deriv of $4u^{1/2} \times$ their deriv of h^3+1 |
| 5 | (ii) | | $dV/dt = 0.4$ $dV/dt = dV/dh \times dh/dt$ $0.4 = 8 \times dh/dt \Rightarrow dh/dt = 0.05$ (m per min) | B1 M1 A1cao [3] | soi o.e. 0.05 or 1/20 | condone r for t any correct chain rule in V, h, t (or r) |
| 6 | (i) | | $2\cos 2y \, dy/dx = 1$ $\Rightarrow dy/dx = 1/(2\cos 2y)$ when $x = 1/2, y = \pi/12, dy/dx = 1/(2\cos(\pi/6)) = 1/\sqrt{3}$ | M1 A1 M1* A1 [4] | $k \cos 2y \, dy/dx = 1$ substituting $y = \pi/12$ *dep 1 st M1 or $\sqrt{3}/3$ | or $dx/dy = k \cos 2y, k \cos 2y dy = dx$ $dy/dx = k \cos 2y$ is M0 isw from correct exact answer |
| 6 | (ii) | | $2y = \arcsin(x - 1)$ $\Rightarrow y = 1/2 \arcsin(x - 1)$ translation of 1 unit in positive x -direction [one-way] stretch s.f. $1/2$ in y -direction | M1 A1 B1 B1 [4] | or $1/2 \sin^{-1}(x - 1)$ or translation $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ not 'shrink', 'squash' etc | allow 'shift', but not 'move', vector only is B0 transformations can be in either order |

| Question | | | Answer | Marks | Guidance | |
|----------|------|--|--|-----------------------------------|--|---|
| 7 | | | $x^{2n} - 1 = (x^n - 1)(x^n + 1)$ one of $2^n - 1$, $2^n + 1$ is divisible by three $2^n - 1$, 2^n and $2^n + 1$ are consecutive integers; one must therefore be divisible by 3; but 2^n is not, so one of the other two is or 2^n is not div by 3, and so has remainder 1 or 2 when divided by 3; if remainder is 1, $2^n - 1$ is div by 3; if remainder is 2, then $2^n + 1$ is div by 3 [so $2^{2n} - 1$ is divisible by 3] | B1 M1 A1 A1 A2 [4] | if justified, correct reason must be given | award notwithstanding false reasoning condone 'factor' for 'multiple' |
| 8 | (i) | | $\frac{dy}{dx} = \frac{(x+4)^{1/2} \cdot 1 - x \cdot \frac{1}{2}(x+4)^{-1/2}}{[(x+4)^{1/2}]^2}$ $= \frac{x+4 - \frac{1}{2}x}{(x+4)^{3/2}} = \frac{\frac{1}{2}x+4}{(x+4)^{3/2}} = \frac{x+8}{2(x+4)^{3/2}} *$ | M1 B1 A1 M1 A1 [5] | quotient rule: $v \times \text{their } u' - u \times \text{their } v'$, and correct denominator $\frac{1}{2} u^{-1/2}$ soi correct expression factoring out $(x+4)^{-1/2}$ o.e. NB AG | or product rule or $-\frac{1}{2} u^{-3/2}$ (PR) PR: $x(-\frac{1}{2})(x+4)^{-3/2} + (x+4)^{-1/2}$ $= (x+4)^{-3/2}(-\frac{1}{2}x + x + 4)$ |
| 8 | (ii) | | [asymptote is] $x = -4$ gradient of tangent at O = $8/(2 \times 4^{3/2}) = \frac{1}{2}$ eqn of tangent is $y = \frac{1}{2}x$ When $x = -4$, $y = -2$, so $(-4, -2)$ | B1 B1 B1 B1 [4] | soi gradient = $\frac{1}{2}$ o.e. e.g. using gradient | but from correct working |

| Question | | | Answer | Marks | Guidance | |
|----------|-------|--|--|-------------------|---|---|
| 8 | (iii) | | let $u = x + 4$, $du = dx$ | B1 | or $dx/du = 1$ | or $v^2 = x+4$, $2v dv/dx = 1$ or $2v dv = dx$ oe e.g. $dv/dx = \frac{1}{2}(x+4)^{-1/2}$ |
| | | | $\int_0^5 \frac{x}{(x+4)^{1/2}} dx = \int_4^9 \frac{u-4}{u^{1/2}} du$ | B1 | $\int \frac{u-4}{u^{1/2}} [du]$ | $\int \frac{v^2-4}{v} 2v [dv]$ |
| | | | $= \int_4^9 (u^{1/2} - 4u^{-1/2}) du$ | B1 | $u^{1/2} - 4u^{-1/2}$ or $u^{1/2} - 4/u^{1/2}$, or $\sqrt{u} - 4/\sqrt{u}$ | $\int (2v^2 - 8) [dv]$ |
| | | | $= \left[\frac{2}{3} u^{3/2} - 8u^{1/2} \right]_4^9$ $= (18 - 24) - (16/3 - 16)$ $= 14/3$ | B1 M1 A1cao | $\left[\frac{2}{3} u^{3/2} - 8u^{1/2} \right]$ o.e. substituting correct limits (upper – lower) | $\left[\frac{2}{3} v^3 - 8v \right]$ 0, 5 for x ; 4,9 for u ; 2,3 for v |
| | | | or (following first 2 marks) let $v = u - 4$, $w' = u^{-1/2}$, $v' = 1$, $w = 2u^{1/2}$ $\int_4^9 (u-4)u^{-1/2} du = \left[2u^{1/2}(u-4) \right]_4^9 - \int_4^9 2u^{1/2} du$ | M1 A1 | | by parts with no substitution: $u = x, u' = 1, v' = (x+4)^{-1/2}, v = 2(x+4)^{1/2}$ M1 $= [2x(x+4)^{1/2}] - \int 2(x+4)^{1/2} A1$ |
| | | | $= \left[2u^{1/2}(u-4) - \frac{4}{3} u^{3/2} \right]_4^9$ $= 14/3$ | A1 A1cao | | $= \left[2x(x+4)^{1/2} - \frac{4}{3}(x+4)^{3/2} \right]_0^5$ A1 $= 14/3$ A1 (so max of 4/6) |
| | | | y- coordinate of Q is $2\frac{1}{2}$ Area of triangle = $\frac{1}{2} \times 5 \times 5/2 = 25/4$ | B1 B1 | (soi) | or $\int_0^5 \frac{1}{2} x dx$ M1 |
| | | | Enclosed area = $25/4 - 14/3 = 1\frac{7}{12}$ | B1 [9] | or $19/12$, or $1.58\dot{3}$ | $= \left[\frac{1}{4} x^2 \right]_0^5 = 25/4$ A1 isw from correct exact answer |

| Question | | | Answer | Marks | Guidance | |
|----------|-------|-----|--|--|--|--|
| 9 | (i) | | $1 + k$ | B1 [1] | | |
| 9 | (ii) | | $f'(x) = 2e^{2x} - 2ke^{-2x}$ $f'(x) = 0 \Rightarrow 2e^{2x} - 2ke^{-2x} = 0$ $\Rightarrow e^{2x} = ke^{-2x}$ $\Rightarrow e^{4x} = k, 4x = \ln k, x = \frac{1}{4} \ln k$ $y = e^{(\frac{1}{2} \ln k)} + ke^{(-\frac{1}{2} \ln k)}$ $= \sqrt{k} + k/\sqrt{k} = 2\sqrt{k}$ | B1 M1 A1 M1 A1cao [5] | their derivative = 0 NB AG substituting $x = \frac{1}{4} \ln k$ into $f(x)$ or $2k^{1/2}$ | |
| 9 | (iii) | | $\text{Area} = \int_0^{\frac{1}{2} \ln k} (e^{2x} + ke^{-2x}) dx$ $= \int_0^{\frac{1}{2} \ln k} (e^{2x} + ke^{-2x}) dx = \left[\frac{1}{2} e^{2x} - \frac{1}{2} ke^{-2x} \right]_0^{\frac{1}{2} \ln k}$ $= \frac{1}{2} k - \frac{1}{2} - \frac{1}{2} + \frac{1}{2} k$ $= k - 1$ | B1 B1 M1 A1 [4] | correct integral and limits (soi) $\left[\frac{1}{2} e^{2x} - \frac{1}{2} ke^{-2x} \right]$ $e^{\ln k} = k$ or $e^{-\ln k} = 1/k$ (soi) | |
| 9 | (iv) | (A) | $g(x) = e^{2(x + \frac{1}{4} \ln k)} + ke^{-2(x + \frac{1}{4} \ln k)}$ $= e^{2x} \cdot e^{\frac{1}{2} \ln k} + k e^{-2x} \cdot e^{-\frac{1}{2} \ln k}$ $= (e^{\ln k})^{1/2} e^{2x} + k \cdot (e^{\ln k})^{-1/2} e^{-2x}$ $= k^{1/2} e^{2x} + k \cdot k^{-1/2} \cdot e^{-2x}$ $= \sqrt{k} (e^{2x} + e^{-2x})$ * | M1 M1 A1 [3] | Substitute $x + \frac{1}{4} \ln k$ for x in $f(x)$ $e^{p+q} = e^p \times e^q$ used NB AG – must show enough working | condone missing brackets e.g. $k^{1/2} e^{2x} + k \cdot k^{-1/2} \cdot e^{-2x}$ |
| 9 | (iv) | (B) | $g(-x) = \sqrt{k} (e^{-2x} + e^{2x})$ $= g(x)$ so g is even | M1 A1 [2] | substituting $-x$ for x must include $g(-x) = g(x)$, and either define an even function or conclude that g is even | condone 'f' used instead of 'g' for M1 not $f(-x) = f(x)$ for A1 |
| 9 | (iv) | (C) | $g(x)$ is symmetrical about the y-axis, and $f(x)$ is $g(x)$ translated $\frac{1}{4} \ln k$ in x -direction so $f(x)$ is symmetrical about $x = \frac{1}{4} \ln k$ | B1 B1 B1 [3] | allow 'shift' or 'move' allow final B1 even if unsupported | or g is f translated $-\frac{1}{4} \ln k$ or incorrectly supported |

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