



**GCE**

**Mathematics (MEI)**

Unit **4757**: Further Applications of Advanced Mathematics

Advanced GCE

**Mark Scheme for June 2016**

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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## Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

**Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand**

- a Annotations should be used whenever appropriate during your marking.

**The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks.** It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

**M**

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

**A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**

Mark for a correct result or statement independent of Method marks.

**E**

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep \*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
  - e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.
- Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

## g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

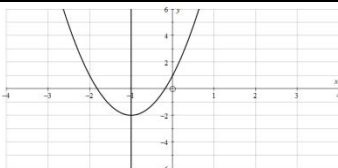
Question		Answer	Marks	Guidance	
1	(i)	$\begin{pmatrix} 3 \\ 4 \\ 8 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -8 \\ 16 \\ -5 \end{pmatrix}$ <p>Distance is <math>\frac{\sqrt{8^2 + 16^2 + 5^2}}{\sqrt{2^2 + 1^2 + 0^2}} = \frac{\sqrt{345}}{\sqrt{5}}</math></p> $= \sqrt{69} \quad (\approx 8.31) \text{ (km)}$	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p>	<p>Appropriate vector product</p> <p>Correctly evaluated</p> <p>Dividing by <math>\sqrt{2^2 + 1^2 + 0^2}</math></p>	<p><i>Sign error in vector product can earn M1A0A1A1</i></p>
	OR	$\begin{pmatrix} 3 + 2\lambda \\ 4 + \lambda \\ 8 \end{pmatrix} \text{ is perpendicular to } \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ $2(3 + 2\lambda) + (4 + \lambda) = 0$ $\lambda = -2; \text{ shortest vector is } \begin{pmatrix} -1 \\ 2 \\ 8 \end{pmatrix}$ <p>Distance is <math>\sqrt{1^2 + 2^2 + 8^2} = \sqrt{69}</math></p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p>		
			[4]		
	(ii)	$\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -8 \\ -4 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 77 \\ 36 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} = -5 \quad \left[ \text{or } \begin{pmatrix} 3 \\ 4 \\ 8 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} \right]$ <p>Distance is <math>\frac{5}{\sqrt{1^2 + 2^2 + 0^2}}</math></p> $= \frac{5}{\sqrt{5}} = \sqrt{5} = 2.236 \dots = 2.24 \text{ (km) (correct to 3 sf)}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>E1</p>	<p>Vector product of directions</p> <p>Correctly evaluated</p> <p>Appropriate scalar product For (-) 5</p> <p>Dividing by <math>\sqrt{1^2 + 2^2 + 0^2}</math></p>	<p><i>Dependent on previous M1</i></p> <p><i>Dependent on M1M1</i></p>
			[6]		

Question			Answer	Marks	Guidance	
	(iii)		$\overrightarrow{AB} = \begin{pmatrix} 80-8\mu \\ 40-4\mu \\ 10-\mu \end{pmatrix} - \begin{pmatrix} 3+2\lambda \\ 4+\lambda \\ 8 \end{pmatrix} \left[ = \begin{pmatrix} 77-2\lambda-8\mu \\ 36-\lambda-4\mu \\ 2-\mu \end{pmatrix} \right]$	B1		
			$\overrightarrow{AB} \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = 0 \quad \text{and} \quad \overrightarrow{AB} \cdot \begin{pmatrix} -8 \\ -4 \\ -1 \end{pmatrix} = 0$ $2(77-2\lambda-8\mu) + (36-\lambda-4\mu) = 0$ $-8(77-2\lambda-8\mu) - 4(36-\lambda-4\mu) - (2-\mu) = 0$ $5\lambda + 20\mu = 190$ $20\lambda + 81\mu = 762$ <p>and hence <math>\lambda = 30, \mu = 2</math></p>	M1  A1  A1		
		OR	$\overrightarrow{AB} \text{ is parallel to } \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$ $36-\lambda-4\mu = -2(77-2\lambda-8\mu)$ $2-\mu = 0$ $5\lambda + 20\mu = 190$ $2-\mu = 0$ <p>and hence <math>\lambda = 30, \mu = 2</math></p>		M1  A1  A1	
		OR	$\overrightarrow{AB} = (\pm) \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$ $77-2\lambda-8\mu = 1$ $36-\lambda-4\mu = -2$ $2-\mu = 0$ $\lambda = 30, \mu = 2$		M1  A1  A1	



Question			Answer	Marks	Guidance	
			<p>Closest points are A(63, 34, 8) and B(64, 32, 8)</p> <p>P is at A at time</p> $t_1 = \frac{\sqrt{5}\lambda}{900} \left[ \text{or } \frac{\sqrt{60^2 + 30^2 + 0^2}}{900} = \frac{\sqrt{4500}}{900} \right] = \frac{\sqrt{5}}{30}$ <p>Q is at B at time <math>t_2 = \frac{9\mu}{270} \left[ \text{or } \frac{\sqrt{16^2 + 8^2 + 2^2}}{270} = \frac{18}{270} \right] = \frac{2}{30}</math></p> <p>These times are different, so the planes are never this close</p>	<p>A1</p> <p>M1</p> <p>E1</p>	<p>Method for finding time</p> <p>Both times correct, and conclusion</p>	
				[7]		

Question			Answer	Marks	Guidance	
	(iv)		$\mathbf{q} = \begin{pmatrix} 80 \\ 40 \\ 10 \end{pmatrix} + \frac{270t}{9} \begin{pmatrix} -8 \\ -4 \\ -1 \end{pmatrix} \quad \left[ = \begin{pmatrix} 80 - 240t \\ 40 - 120t \\ 10 - 30t \end{pmatrix} \right]$ $\mathbf{r} = \begin{pmatrix} 29 \\ 19 \\ 5.5 \end{pmatrix} + \frac{285t}{19} \begin{pmatrix} 18 \\ 6 \\ 1 \end{pmatrix} \quad \left[ = \begin{pmatrix} 29 + 270t \\ 19 + 90t \\ 5.5 + 15t \end{pmatrix} \right]$ $80 - 240t = 29 + 270t$ <p>Q, R will collide if <math>40 - 120t = 19 + 90t</math> <math>10 - 30t = 5.5 + 15t</math></p> <p>All three equations have solution <math>t = 0.1</math> Planes would collide; so R must alter course or speed</p>	M1 A1  A1  M1  A1 A1 E1	Speed and unit direction vectors       One equation sufficient for M1  For $t = 0.1$ obtained Shown to satisfy all three Correctly shown	         Point of collision is (56, 28, 7)
		OR	$80 - 8\mu = 29 + 18\nu$ <p>Paths intersect if <math>40 - 4\mu = 19 + 6\nu</math> <math>10 - \mu = 5.5 + \nu</math></p> $\mu = 3, \nu = 1.5$ <p>All equations are satisfied, so paths intersect [ at X (56, 28, 7) ]</p> $\text{Q is at X at time } t = \frac{9\mu}{270} \left[ \text{or } \frac{\sqrt{24^2 + 12^2 + 3^2}}{270} = \frac{27}{270} \right] = 0.1$ $\text{R is at X at time } t = \frac{19\nu}{285} \left[ \text{or } \frac{\sqrt{27^2 + 9^2 + 1.5^2}}{285} = \frac{28.5}{285} \right] = 0.1$ <p>Planes would collide; so R must alter course or speed</p>		M1 OR A1 Two correct equations  A1  A1 <i>Must check third equation</i>  M1 Method for finding time A1 For $t = 0.1$  E1 Correctly shown	$\Delta = \begin{pmatrix} -51 \\ -21 \\ -4.5 \end{pmatrix} \cdot \left[ \begin{pmatrix} -8 \\ -4 \\ -1 \end{pmatrix} \times \begin{pmatrix} 18 \\ 6 \\ 1 \end{pmatrix} \right]$ $\begin{pmatrix} -8 \\ -4 \\ -1 \end{pmatrix} \times \begin{pmatrix} 18 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -10 \\ 24 \end{pmatrix}$ <p><math>\Delta = 0</math>, so the paths intersect</p>   <

Question		Answer	Marks	Guidance	
2	(i)	 $y = 1 \Rightarrow z = 3x^2 + 6x + 1 \quad [= 3(x+1)^2 - 2]$ <p>So line of symmetry is <math>\mathbf{r} = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}</math></p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1B1</p>	<p>Correct shape</p> <p>Minimum in third quadrant and positive intercept on z-axis</p> <p>For <math>\begin{pmatrix} -1 \\ 1 \\ . \end{pmatrix}</math> and <math>\lambda \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}</math></p>	
			[5]		
2	(ii)	<p>We require <math>\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = 0</math></p> <p><math>\frac{\partial z}{\partial x} = 6x + 6y = 0 \quad [\Rightarrow y = -x]</math></p> <p><math>\frac{\partial z}{\partial y} = 6x + 3y^2 = 0</math> and hence <math>y^2 = 2y</math></p> <p><math>y = 0, 2; \quad x = 0, -2; \quad z = 0, -4</math></p> <p>Stationary points are <math>(0, 0, 0)</math> and <math>(-2, 2, -4)</math></p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>E1</p>	<p>For either equation</p> <p>Correct equation in y or x</p> <p><i>Some working required for -4</i></p>	<p>Or <math>6x + 3x^2 = 0</math></p>
			[4]		

Question			Answer	Marks	Guidance	
	(iii)		<p>For <math>x = -2 + h</math>, <math>y = 2 + k</math>, <math>z = \lambda</math></p> $\lambda = 3(-2 + h)^2 + 6(-2 + h)(2 + k) + (2 + k)^3$ $= 12 - 12h + 3h^2 + 6hk + 12h - 12k - 24$ $+ 8 + 12k + 6k^2 + k^3$ $= -4 + 3h^2 + 6hk + 6k^2 + k^3$ $= -4 + 3(h + k)^2 + k^3 + 3k^2$ $= -4 + 3(h + k)^2 + k^2(k + 3)$ <p>Since <math>3(h + k)^2 &gt; 0</math> and <math>k^2(k + 3) &gt; 0</math> for small <math>k</math>  <math>\lambda &gt; -4</math> for all small values of <math>h</math> and <math>k</math>  so P is a minimum.</p>	<p>M1 A1</p> <p>E1</p> <p>M1</p> <p>E1</p>	<p>Substitution</p> <p><i>M0 for numerical work</i></p> <p>Must mention small <math>k</math> or <math>k &gt; -3</math></p>	
				[5]		
	(iv)		<p>For small <math>x</math> and <math>y</math>, <math>z</math> can be positive or negative  If <math>x = 0</math> and <math>y &gt; 0</math>, then <math>z &gt; 0</math>  If <math>x = 0</math> and <math>y &lt; 0</math>, then <math>z &lt; 0</math>  Hence O is neither a maximum nor a minimum</p>	<p>M1</p> <p>A1</p> <p>E1</p>	<p>(Numerical demonstration can earn M1A0E0)  Correct argument which applies arbitrarily close to O  Correctly shown</p>	<p>When <math>x = 0</math>, <math>z = y^3</math> which has a point of inflection</p>
				[3]		
	(v)		<p>We require <math>\frac{\partial z}{\partial x} = 18</math>, <math>\frac{\partial z}{\partial y} = 18</math></p> $6x + 6y = 18, 6x + 3y^2 = 18$ $2(3 - y) + y^2 = 6$ <p>Points are (3, 0, 27) and (1, 2, 23)</p> $18x + 18y - z = d$ <p>So <math>d = 27, 31</math></p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>Allow -18 for M1</p> <p>Obtaining equation for <math>y</math> or <math>x</math></p> <p>or <math>2x + (3 - x)^2 = 6</math>  Obtaining values of <math>x, y, z</math></p>	
				[7]		

Question			Answer	Marks	Guidance	
3	(i)		When $x = 0$ , $t = 0, \pm \frac{1}{\sqrt{3}}$ $y = 1, y = 2$	E1 B1	For both	
				[2]		
	(ii)		$\dot{x} = 1 - 9t^2, \dot{y} = 6t \quad \ddot{x} = -18t, \ddot{y} = 6$ $\rho = \frac{\left((1-9t^2)^2 + 36t^2\right)^{3/2}}{6(1-9t^2) + 108t^2} = \frac{(1+9t^2)^3}{6(1+9t^2)} = \frac{(1+9t^2)^2}{6}$ When $t = \frac{1}{\sqrt{3}}, \rho = \frac{16}{6} = \frac{8}{3}$ $\tan \psi = \frac{dy}{dx} = \frac{6t}{1-9t^2}$ $\sin \psi = \frac{6t}{1+9t^2}, \cos \psi = \frac{1-9t^2}{1+9t^2}$ Centre of curvature is at $\left(0 - \frac{8}{3} \times \frac{\sqrt{3}}{2}, 2 - \frac{8}{3} \times \frac{1}{2}\right)$ i.e. $\left(-\frac{4\sqrt{3}}{3}, \frac{2}{3}\right)$	B1 M1 A1  A1 M1 A1 M1 A1  M1  A1A1	All 4 soi Use of formula for $\rho$ or $\kappa$ Unsimplified  or unit normal is $\begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$	$\dot{x} = -2, \dot{y} = \frac{6}{\sqrt{3}} \quad \ddot{x} = -\frac{18}{\sqrt{3}}, \ddot{y} = 6$ $\rho = \frac{(4+12)^{3/2}}{-12+36}$  $\tan \psi = \frac{dy}{dx} = -\sqrt{3}$ $\sin \psi = \frac{\sqrt{3}}{2}, \cos \psi = -\frac{1}{2}$
				[11]		
	(iii)		$\dot{x}^2 + \dot{y}^2 = (1-9t^2)^2 + (6t)^2 = (1+9t^2)^2$ $s = \int_0^{1/\sqrt{3}} (1+9t^2) dt$ $= \left[t + 3t^3\right]_0^{1/\sqrt{3}}$ $= \frac{2}{\sqrt{3}} = \frac{2}{3}\sqrt{3}$	M1A1  M1  A1 A1	Soi  Limits not required  Including limits	
				[5]		

Question			Answer	Marks	Guidance	
	(iv)		$S = 2\pi \int_0^{\frac{1}{\sqrt{3}}} x \, ds = 2\pi \int_0^{\frac{1}{\sqrt{3}}} (t - 3t^3)(1 + 9t^2) dt$ $= 2\pi \int_0^{\frac{1}{\sqrt{3}}} (t + 6t^3 - 27t^5) dt = 2\pi \left[ \frac{t^2}{2} + \frac{3}{2}t^4 - \frac{9}{2}t^6 \right]_0^{\frac{1}{\sqrt{3}}}$ $= 2\pi \left( \frac{1}{6} + \frac{1}{6} - \frac{1}{6} \right) = \frac{\pi}{3}$	M1 M1 A1  M1 A1  E1	Correct formula Integral in terms of $t$ <i>Including limits</i>  Expand and integrate <i>Including limits</i>  <i>Intermediate step required</i>	
				[6]		

Question			Answer					Marks	Guidance											
4	(a)	(i)	$3*(9*11) = 3*3 = 9$ $(3*9)*11 = 11*11 = 9$  Construction of group table (or otherwise):  It shows closure, the identity is 1 each element has an inverse $3^{-1} = 11, 9^{-1} = 9, 11^{-1} = 3, 1^{-1} = 1$					B1 B1  B1  B1 B1  B1		Group table $\begin{matrix} & 1 & 3 & 9 & 11 \\ 1 & \left( \begin{matrix} 1 & 3 & 9 & 11 \\ 3 & 9 & 11 & 1 \\ 9 & 9 & 11 & 1 & 3 \\ 11 & 11 & 1 & 3 & 9 \end{matrix} \right) \end{matrix}$										
								[6]												
		(ii)	<table><tr><td>Element</td><td>1</td><td>3</td><td>9</td><td>11</td></tr><tr><td>Order</td><td>1</td><td>4</td><td>2</td><td>4</td></tr></table>	Element	1	3	9	11	Order	1	4	2	4					B2	-1 each error	
Element	1	3	9	11																
Order	1	4	2	4																
								[2]												
		(iii)	{ 1 } { 1,9 } { 1,3,9,11 }					B1	Condone omission of trivial subgroups	B0 if any extras										
								[1]												
		(iv)	e.g. $3^2 = 9, 3^3 = 11, 3^4 = 1$ 3 generates the group and so it is cyclic					E1												
								[1]												
	(b)		Composition table: $\begin{matrix} & e & a & b & ab \\ e & \left( \begin{matrix} e & a & b & ab \\ a & a & e & ab & b \\ b & b & ab & e & a \\ ab & ab & b & a & e \end{matrix} \right) \end{matrix}$ All elements are self-inverse, and so no element generates the group					B3          E1	-1 each error											
								[4]												

Question			Answer	Marks	Guidance	
	(c)		In group G all elements are self-inverse i.e. $X^2 = I$ , $Y^2 = I$ and $Z^2 = I$ So this group is isomorphic to the group in (b) e.g. $I \leftrightarrow e$ $X \leftrightarrow a$ $Y \leftrightarrow b$ $Z \leftrightarrow ab$	M1 A1A1 A1 B1B1	Correctly shown	
				[6]		
	(d)		One of the elements needs to be the identity element. It is neither p nor q for otherwise $p \circ q = p$ (or q) It is neither r nor s, for otherwise $p \circ q = q \circ p = r$ (or s) So there is no identity element and so not a group	M1 A1 A1 E1		
				[4]		



Question			Answer	Marks	Guidance	
5	(i)		$\begin{pmatrix} 0.75 & 0 & p \\ 0.125 & 0.5 & \frac{1-p}{2} \\ 0.125 & 0.5 & \frac{1-p}{2} \end{pmatrix}$	B1 M1  A1	1st two columns Making 3rd column sum to 1	
				[3]		
	(ii)		$\begin{pmatrix} 0.75 & 0 & p \\ 0.125 & 0.5 & \frac{1-p}{2} \\ 0.125 & 0.5 & \frac{1-p}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$ $0.75 + p = 1$ $p = 0.25$	M1 A1    A1 A1	Equilibrium probs Equation    <i>Correct equation implies            M1A1A1            Just answer: B4</i>	
				[4]		
	(iii)		$P(\text{A on day 5}) = \begin{pmatrix} 0.75 & 0 & 0.25 \\ 0.125 & 0.5 & 0.375 \\ 0.125 & 0.5 & 0.375 \end{pmatrix}^4 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $= \begin{pmatrix} 0.435059 & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{pmatrix}$ $= 0.435$	M1   A1  A1	For power 4   At least one right, soi  <i>Just answer: B3</i>	
				[3]		

Question			Answer	Marks	Guidance	
	(iv)		$\mathbf{P} = \begin{pmatrix} 0.75 & 0 & 0.4 \\ 0.125 & 0.5 & 0.3 \\ 0.125 & 0.5 & 0.3 \end{pmatrix}, \quad \mathbf{P}^3 = \begin{pmatrix} 0.536875 & 0.31 & 0.431 \\ 0.231563 & 0.345 & 0.2845 \\ 0.231563 & 0.345 & 0.2845 \end{pmatrix}$ $\mathbf{P}^3 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.536875 \\ 0.231563 \\ 0.231563 \end{pmatrix}$ $p = 0.536875 \times 0.536875 + 0.345 \times 0.2315625 + 0.2845 \times 0.2315625$ $= 0.434(003)\dots$	M1  M1 A1  M1 A1	For using $\mathbf{P}^3$  First column of $\mathbf{P}^3$	
				[5]		
	(v)		P(from A to A) = 0.75 so $\alpha = 0.75$ Expected number is $\frac{\alpha}{1-\alpha} = \frac{0.75}{0.25} = 3$	B1 M1 A1	Using $\frac{\alpha}{1-\alpha}$	
				[3]		
	(vi)		$\begin{pmatrix} 0 & 0.5 & 0.5 \\ 0 & 0 & 0.5 \\ 1 & 0.5 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $0.5y + 0.5z = x, \quad 0.5z = y, \quad x + 0.5y = z$ $x + y + z = 1$ $x = \frac{1}{3}, y = \frac{2}{9}, z = \frac{4}{9}$	B1  M1 A1  M1  A2	New transition matrix      -1 each error	
		OR	New transition matrix Considering a high power (at least 20) $P(A) = 0.333, P(B) = 0.222, P(C) = 0.444$		B1 M2 Give M1 for at least 10 A1A1A1	
				[6]		

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