

OCR

Oxford Cambridge and RSA

Tuesday 20 June 2017 – Afternoon

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Differentiate $\frac{1}{(5-2x^3)^2}$. [3]

2 The function $f(x)$ is defined by $f(x) = |x|$, for $-1 \leq x \leq 1$.
Sketch the graph of $y = g(x)$, where $g(x) = 2 - 2f(x)$. [3]

3 Functions $f(x)$ and $g(x)$, each defined for $-1 < x < 1$, are given by $f(x) = \ln(1-x)$ and $g(x) = x^2$.
(i) Find $f^{-1}(x)$ and state its domain and range. [4]
(ii) Show that $f(x) + f(-x) = fg(x)$. [3]

4 A curve has equation $3x^{\frac{2}{3}} + 2y^{\frac{1}{3}} = 7$.
(i) By differentiating implicitly, find $\frac{dy}{dx}$ in terms of x and y . [3]
(ii) Hence find the gradient of the curve at the point with coordinates $(1, 8)$. [2]

5 A liquid is being heated. At time t minutes after heating starts, its temperature, $\theta^\circ\text{C}$, is modelled by the equation

$$\theta = 10.5 + 69.5(1 - e^{-kt}),$$

where k is a positive constant. The boiling point of the liquid is the value approached by θ as t tends to infinity.

(i) Write down the initial temperature and the boiling point of the liquid. [2]
(ii) After being heated for one minute, the liquid has a temperature of 30°C . Find k . [3]
(iii) Find how long it takes from the start of the heating until the temperature is within 1°C of the boiling point. Give your answer to the nearest minute. [3]

6 You are given that the sum of the interior angles of a polygon with n sides is $180(n-2)^\circ$. Using this result, or otherwise, prove that the interior angle of a regular polygon cannot be 155° . [3]

7 The equation of a curve is $y = \arcsin \frac{1}{2}x$.
(i) Express each of x and $\frac{dx}{dy}$ in terms of y . [2]
(ii) A point is moving on the curve, and has coordinates (x, y) at time t . When $x = 1$, the value of $\frac{dx}{dt}$ is 2.
Find the exact value of $\frac{dy}{dt}$ at this instant. [5]

Section B (36 marks)

- 8 Fig. 8 shows part of the curve $y = \frac{\cos x}{2 - \sin x}$. The curve intersects the x - and y -axes at A and C respectively, and has a turning point at B.

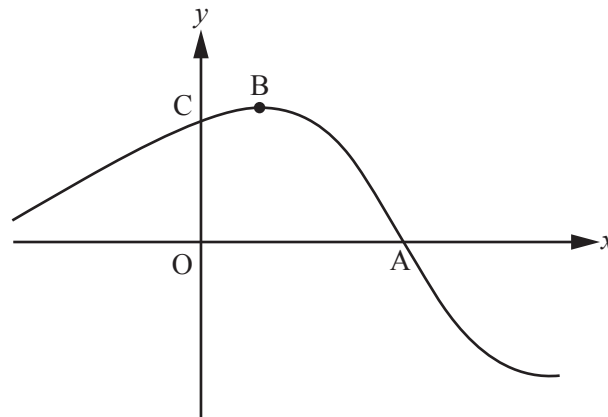


Fig. 8

- (i) Write down the coordinates of A and C. [2]
- (ii) Find $\frac{dy}{dx}$ and the exact coordinates of B. [7]
- (iii) (A) Using integration by substitution, or otherwise, find the exact area of the region enclosed by the curve, the y -axis and the positive x -axis. [4]
- (B) The line $x = k$ divides this region into two parts of equal area. Show that $k = \arcsin(2 - \sqrt{2})$. [5]
- 9 A curve has equation $y = f(x)$, where $f(x) = x^3 e^{-x^2}$.
- (i) Show that $f(x)$ is an odd function, and interpret this result in terms of the graph of the curve $y = f(x)$. [3]
- (ii) Find the coordinates of the stationary points of the curve. Give answers correct to 2 decimal places where appropriate. [7]
- (iii) Sketch the curve for $-2 \leq x \leq 2$. [2]
- (iv) (A) Show, using the substitution $t = x^2$, that $\int f(x) dx$ may be expressed as $\int k t e^{-t} dt$, where k is a constant to be determined. [2]
- (B) Hence find the exact area of the region enclosed by the curve $y = f(x)$, the positive x -axis and the line $x = 2$. [4]

END OF QUESTION PAPER

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