



Oxford Cambridge and RSA

**Friday 23 June 2017 – Morning**

**A2 GCE MATHEMATICS (MEI)**

**4754/01A Applications of Advanced Mathematics (C4) Paper A**

**QUESTION PAPER**



Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration: 1 hour 30 minutes**

**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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## Section A (36 marks)

1 (i) Express  $\frac{5-x}{(2-x)(1+x)}$  in partial fractions. [3]

(ii) Hence or otherwise find the first 3 terms of the binomial expansion of  $\frac{5-x}{(2-x)(1+x)}$  in ascending powers of  $x$ . [5]

2 The equation of a line is  $\mathbf{r} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$  and the equation of a plane is  $3x + 4y - z = 17$ .

(i) Find the coordinates of the point of intersection of the line and the plane. [4]

(ii) Find the acute angle between the line and the normal to the plane. [4]

3 Fig. 3 shows the curve  $y = \sqrt{1 + e^{2x}}$ .

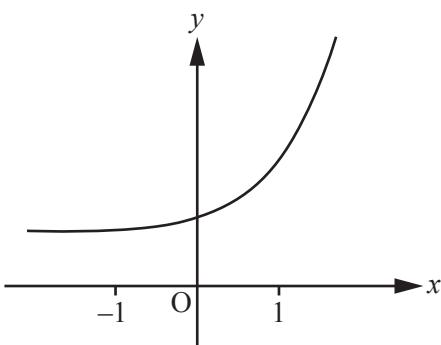


Fig. 3

The value of  $\int_{-1}^1 \sqrt{1 + e^{2x}} dx$  is to be estimated using the trapezium rule.  $T_2$  and  $T_4$  are the estimates obtained from the trapezium rule using 2 strips and 4 strips respectively.

(i) Explain whether  $T_4$  is greater or less than  $T_2$ . [2]

(ii) Evaluate  $T_4$ , giving your answer to 3 significant figures. [4]

4 Vectors  $\mathbf{u}$  and  $\mathbf{v}$  are given by  $\mathbf{u} = \mathbf{i} - 7\mathbf{j} - 2\mathbf{k}$  and  $\mathbf{v} = a\mathbf{i} + b\mathbf{j} + 5\mathbf{k}$ , where  $a$  and  $b$  are constants.

Find  $a$  and  $b$  given that the magnitude of  $\mathbf{v}$  is  $\sqrt{27}$  and that  $\mathbf{u}$  and  $\mathbf{v}$  are perpendicular. [6]

5 Solve the equation  $4 \tan \theta \tan 2\theta = 1$ , for  $0^\circ < \theta < 180^\circ$ . [4]

6 The number of bacteria in a population at time  $t$  is denoted by  $P$ . The rate of increase of  $P$  is proportional to the square root of  $P$ .

(i) Write down a differential equation relating  $P$ , the time  $t$ , and a constant of proportionality  $k$ . [1]

(ii) Verify that  $P = (A + Bt)^2$ , where  $A$  and  $B$  are constants, satisfies the differential equation, and find  $k$  in terms of  $B$ . [3]

## Section B (36 marks)

7 The curve shown in Fig. 7 passes through the origin and satisfies the differential equation

$$\frac{dy}{dx} = \frac{9x}{4(y+3)}.$$

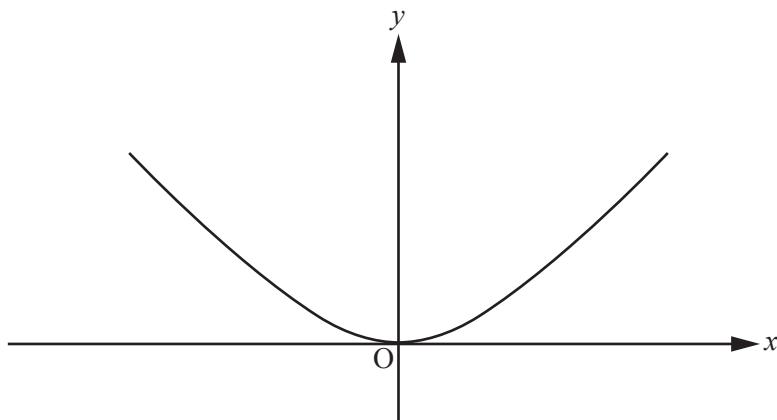


Fig. 7

(i) Show by integration that the equation of the curve is  $9x^2 - 4y^2 - 24y = 0$ . [5]

The finite region bounded by the curve and the line  $y = 2$  is rotated through  $180^\circ$  about the  $y$ -axis.

(ii) Find the volume of the solid of revolution generated, giving your answer as an exact multiple of  $\pi$ . [4]

(iii) Use the substitutions  $x = 2 \tan \theta$  and  $y = 3(\sec \theta - 1)$

(A) to verify that  $9x^2 - 4y^2 - 24y = 0$ ,

(B) to show that  $\frac{9x}{4(y+3)}$  can be expressed as  $k \sin \theta$ , where  $k$  is a constant to be found.

Hence find the exact gradient of the curve at the point with  $x$ -coordinate 2. [9]

8 Fig. 8 shows the curve with parametric equations

$$x = \cos 2\theta, y = \cos \theta + 2 \sin \theta, \text{ for } -\pi < \theta \leq \pi.$$

The curve intersects the  $x$ -axis at A, and the points B and C have maximum  $x$ - and  $y$ -coordinates respectively.

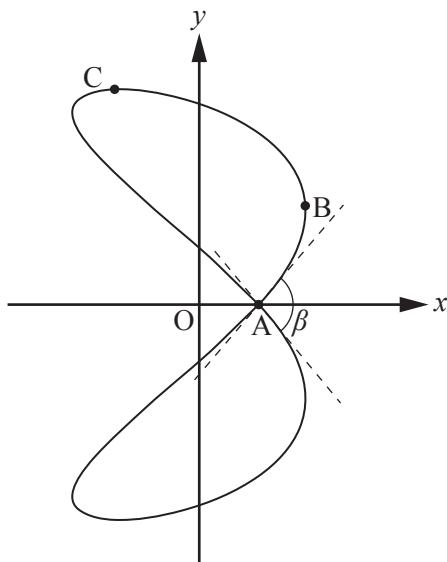


Fig. 8

(i) Find the value of  $\theta$  corresponding to the point B. Hence find the coordinates of the point B. [3]

(ii) Express  $\cos \theta + 2 \sin \theta$  in the form  $R \cos(\theta - \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{1}{2}\pi$ . [5]

(iii) Hence find the coordinates of the points A and C. [5]

The angle  $\beta$  is the angle between the tangents to the curve at A.

(iv) Find  $\frac{dy}{dx}$  in terms of  $\theta$ . Hence, assuming the scales of the  $x$ - and  $y$ -axes are equal, find  $\beta$ , giving the answer in radians correct to 2 decimal places. [You may assume the curve is symmetrical about the  $x$ -axis.] [5]

**END OF QUESTION PAPER**

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