OCR
Oxford Cambridge and RSA

Monday 26 June 2017 – Afternoon

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (54 marks)

1 **(a)** **(i)** By differentiating the equation $a\tan y = x$ show that

$$\int \frac{1}{a^2+x^2}\,\mathrm{d}x = \frac{1}{a}\arctan\left(\frac{x}{a}\right)+c\ .$$

[3]

The cartesian equation of an ellipse is $\frac{x^2}{4}+\frac{y^2}{9}=1$.

(ii) Show that the polar equation of the ellipse may be written in the form

$$r^2 = \frac{36\sec^2\theta}{9+4\tan^2\theta}\ .$$

[3]

(iii) By using the substitution $3u = 2\tan\theta$ show that the area enclosed by the ellipse and the lines $\theta = 0$ and $\theta = \frac{\pi}{4}$ is $3\arctan\left(\frac{2}{3}\right)$. **[7]**

(b) Obtain the first three terms of the Maclaurin series for $\mathrm{f}(x)$, where $\mathrm{f}(x) = \arctan(1+x)$. **[5]**

2 **(a)** The infinite series C and S are defined as follows.

$$C = -\frac{1}{2}\cos\theta + \frac{1}{4}\cos 2\theta - \frac{1}{8}\cos 3\theta + \ldots$$
$$S = -\frac{1}{2}\sin\theta + \frac{1}{4}\sin 2\theta - \frac{1}{8}\sin 3\theta + \ldots$$

By considering $C + \mathrm{j}S$, show that

$$S = \frac{-2\sin\theta}{5 + 4\cos\theta}.$$

Find a corresponding expression for C. **[9]**

(b) In an Argand diagram, O is the origin and points A and B are represented by the complex conjugate pair z_1 and z_2 respectively, where $0 < \arg z_1 < \frac{\pi}{2}$. The triangle OAB has side OA of length a.

(i) Show the above information on an Argand diagram. **[1]**

(ii) Show that $z_1 z_2$ is real, giving its value in terms of a. **[2]**

Triangle OAB is rotated anti-clockwise about the origin through γ radians, where $0 < \gamma < 2\pi$, and then enlarged through the origin with scale factor 3. The resulting new positions of A and B are represented by the complex numbers z_3 and z_4 respectively, where z_3 and z_4 form another complex conjugate pair.

(iii) State the value of γ. **[1]**

(iv) Find, in polar form (modulus-argument form), the complex number $\frac{z_3}{z_1}$. **[2]**

(v) Given that, in the original triangle OAB, AB also has length a, find the complex number $\frac{z_1}{z_4}$, giving your answer in the form $x + \mathrm{j}y$, where x and y are exact real numbers. **[3]**

3 **(a)** You are given the matrix $\mathbf{M} = \begin{pmatrix} k & 2 & 1 \\ 3 & -1 & 2 \\ 1 & 2 & -2 \end{pmatrix}$.

(i) Find the value of k for which $\mathbf{M}$ does not have an inverse. **[3]**

(ii) Find $\mathbf{M}^{-1}$ in terms of k. **[4]**

(b) The matrix $\mathbf{Q}$ is given by $\mathbf{Q} = \begin{pmatrix} 3 & 3 \\ 4 & 7 \end{pmatrix}$.

(i) Find the eigenvalues and corresponding eigenvectors of $\mathbf{Q}$. **[5]**

(ii) State a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{Q} = \mathbf{PDP}^{-1}$. **[2]**

(iii) Show that, for $n \geqslant 1$, $\mathbf{Q}^n = \frac{1}{8}\begin{pmatrix} 6 + 2\varphi & 3\varphi - 3 \\ 4\varphi - 4 & 6\varphi + 2 \end{pmatrix}$, where $\varphi = 9^n$. **[4]**

Section B (18 marks)

4 **(i)** Prove, from definitions involving exponentials, that $\operatorname{sech}^2 x + \tanh^2 x = 1$. **[4]**

(ii) Prove that

$$\operatorname{artanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right).$$

State the set of values of x for which this is valid. **[5]**

(iii) Solve the equation

$$3(\tanh^2 x - \operatorname{sech}^2 x) = \tanh x - 2 ,$$

giving your answers in an exact logarithmic form. **[5]**

(iv) Find the exact value of

$$\int_{\operatorname{arsinh} 2}^{\operatorname{arsinh} 3} \frac{1}{\tanh x - \operatorname{sech} x}\, dx .$$

[4]

END OF QUESTION PAPER