



Oxford Cambridge and RSA

**Monday 26 June 2017 – Afternoon**

**A2 GCE MATHEMATICS (MEI)**

**4756/01 Further Methods for Advanced Mathematics (FP2)**

**QUESTION PAPER**



Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration: 1 hour 30 minutes**

**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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## Section A (54 marks)

1 (a) (i) By differentiating the equation  $a \tan y = x$  show that

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + c. \quad [3]$$

The cartesian equation of an ellipse is  $\frac{x^2}{4} + \frac{y^2}{9} = 1$ .

(ii) Show that the polar equation of the ellipse may be written in the form

$$r^2 = \frac{36 \sec^2 \theta}{9 + 4 \tan^2 \theta}. \quad [3]$$

(iii) By using the substitution  $3u = 2 \tan \theta$  show that the area enclosed by the ellipse and the lines  $\theta = 0$  and  $\theta = \frac{\pi}{4}$  is  $3\arctan\left(\frac{2}{3}\right)$ . [7]

(b) Obtain the first three terms of the Maclaurin series for  $f(x)$ , where  $f(x) = \arctan(1+x)$ . [5]

2 (a) The infinite series  $C$  and  $S$  are defined as follows.

$$C = -\frac{1}{2} \cos \theta + \frac{1}{4} \cos 2\theta - \frac{1}{8} \cos 3\theta + \dots$$

$$S = -\frac{1}{2} \sin \theta + \frac{1}{4} \sin 2\theta - \frac{1}{8} \sin 3\theta + \dots$$

By considering  $C + jS$ , show that

$$S = \frac{-2 \sin \theta}{5 + 4 \cos \theta}.$$

Find a corresponding expression for  $C$ . [9]

(b) In an Argand diagram,  $O$  is the origin and points  $A$  and  $B$  are represented by the complex conjugate pair  $z_1$  and  $z_2$  respectively, where  $0 < \arg z_1 < \frac{\pi}{2}$ . The triangle  $OAB$  has side  $OA$  of length  $a$ .

(i) Show the above information on an Argand diagram. [1]

(ii) Show that  $z_1 z_2$  is real, giving its value in terms of  $a$ . [2]

Triangle  $OAB$  is rotated anti-clockwise about the origin through  $\gamma$  radians, where  $0 < \gamma < 2\pi$ , and then enlarged through the origin with scale factor 3. The resulting new positions of  $A$  and  $B$  are represented by the complex numbers  $z_3$  and  $z_4$  respectively, where  $z_3$  and  $z_4$  form another complex conjugate pair.

(iii) State the value of  $\gamma$ . [1]

(iv) Find, in polar form (modulus-argument form), the complex number  $\frac{z_3}{z_1}$ . [2]

(v) Given that, in the original triangle  $OAB$ ,  $AB$  also has length  $a$ , find the complex number  $\frac{z_1}{z_4}$ , giving your answer in the form  $x + jy$ , where  $x$  and  $y$  are exact real numbers. [3]

3 (a) You are given the matrix  $\mathbf{M} = \begin{pmatrix} k & 2 & 1 \\ 3 & -1 & 2 \\ 1 & 2 & -2 \end{pmatrix}$ .

(i) Find the value of  $k$  for which  $\mathbf{M}$  does not have an inverse. [3]

(ii) Find  $\mathbf{M}^{-1}$  in terms of  $k$ . [4]

(b) The matrix  $\mathbf{Q}$  is given by  $\mathbf{Q} = \begin{pmatrix} 3 & 3 \\ 4 & 7 \end{pmatrix}$ .

(i) Find the eigenvalues and corresponding eigenvectors of  $\mathbf{Q}$ . [5]

(ii) State a matrix  $\mathbf{P}$  and a diagonal matrix  $\mathbf{D}$  such that  $\mathbf{Q} = \mathbf{PDP}^{-1}$ . [2]

(iii) Show that, for  $n \geq 1$ ,  $\mathbf{Q}^n = \frac{1}{8} \begin{pmatrix} (6+2\varphi)^n & 3\varphi^n - 3 \\ (4\varphi - 4)^n & 6\varphi^n + 2 \end{pmatrix}$ , where  $\varphi = 9^n$ . [4]

## Section B (18 marks)

4 (i) Prove, from definitions involving exponentials, that  $\operatorname{sech}^2 x + \tanh^2 x = 1$ . [4]

(ii) Prove that

$$\operatorname{artanh} x = \frac{1}{2} \ln \left( \frac{1+x}{1-x} \right).$$

State the set of values of  $x$  for which this is valid. [5]

(iii) Solve the equation

$$3(\tanh^2 x - \operatorname{sech}^2 x) = \tanh x - 2,$$

giving your answers in an exact logarithmic form. [5]

(iv) Find the exact value of

$$\int_{\operatorname{arsinh} 2}^{\operatorname{arsinh} 3} \frac{1}{\tanh x - \operatorname{sech} x} dx.$$

[4]

**END OF QUESTION PAPER**



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