

OCR

Oxford Cambridge and RSA

Monday 26 June 2017 – Afternoon

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

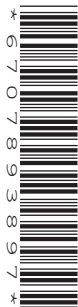
OCR supplied materials:

- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Section A (54 marks)

- 1 (a) (i) By differentiating the equation $a \tan y = x$ show that

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + c . \quad [3]$$

The cartesian equation of an ellipse is $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

- (ii) Show that the polar equation of the ellipse may be written in the form

$$r^2 = \frac{36 \sec^2 \theta}{9 + 4 \tan^2 \theta} . \quad [3]$$

- (iii) By using the substitution $3u = 2 \tan \theta$ show that the area enclosed by the ellipse and the lines $\theta = 0$ and $\theta = \frac{\pi}{4}$ is $3 \arctan\left(\frac{2}{3}\right)$. [7]

- (b) Obtain the first three terms of the Maclaurin series for $f(x)$, where $f(x) = \arctan(1 + x)$. [5]

- 2 (a) The infinite series C and S are defined as follows.

$$C = -\frac{1}{2}\cos\theta + \frac{1}{4}\cos 2\theta - \frac{1}{8}\cos 3\theta + \dots$$

$$S = -\frac{1}{2}\sin\theta + \frac{1}{4}\sin 2\theta - \frac{1}{8}\sin 3\theta + \dots$$

By considering $C + jS$, show that

$$S = \frac{-2\sin\theta}{5+4\cos\theta}.$$

Find a corresponding expression for C .

[9]

- (b) In an Argand diagram, O is the origin and points A and B are represented by the complex conjugate pair z_1 and z_2 respectively, where $0 < \arg z_1 < \frac{\pi}{2}$. The triangle OAB has side OA of length a .

(i) Show the above information on an Argand diagram.

[1]

(ii) Show that $z_1 z_2$ is real, giving its value in terms of a .

[2]

Triangle OAB is rotated anti-clockwise about the origin through γ radians, where $0 < \gamma < 2\pi$, and then enlarged through the origin with scale factor 3. The resulting new positions of A and B are represented by the complex numbers z_3 and z_4 respectively, where z_3 and z_4 form another complex conjugate pair.

(iii) State the value of γ .

[1]

(iv) Find, in polar form (modulus-argument form), the complex number $\frac{z_3}{z_1}$.

[2]

(v) Given that, in the original triangle OAB , AB also has length a , find the complex number $\frac{z_1}{z_4}$, giving your answer in the form $x + jy$, where x and y are exact real numbers.

[3]

- 3 (a) You are given the matrix $\mathbf{M} = \begin{pmatrix} k & 2 & 1 \\ 3 & -1 & 2 \\ 1 & 2 & -2 \end{pmatrix}$.

(i) Find the value of k for which \mathbf{M} does not have an inverse.

[3]

(ii) Find \mathbf{M}^{-1} in terms of k .

[4]

- (b) The matrix \mathbf{Q} is given by $\mathbf{Q} = \begin{pmatrix} 3 & 3 \\ 4 & 7 \end{pmatrix}$.

(i) Find the eigenvalues and corresponding eigenvectors of \mathbf{Q} .

[5]

(ii) State a matrix \mathbf{P} and a diagonal matrix \mathbf{D} such that $\mathbf{Q} = \mathbf{PDP}^{-1}$.

[2]

(iii) Show that, for $n \geq 1$, $\mathbf{Q}^n = \frac{1}{8} \begin{pmatrix} 6+2\varphi & 3\varphi-3 \\ 4\varphi-4 & 6\varphi+2 \end{pmatrix}$, where $\varphi = 9^n$.

[4]

Section B (18 marks)

- 4 (i) Prove, from definitions involving exponentials, that $\operatorname{sech}^2 x + \tanh^2 x = 1$. [4]

- (ii) Prove that

$$\operatorname{artanh} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right).$$

State the set of values of x for which this is valid. [5]

- (iii) Solve the equation

$$3(\tanh^2 x - \operatorname{sech}^2 x) = \tanh x - 2,$$

giving your answers in an exact logarithmic form. [5]

- (iv) Find the exact value of

$$\int_{\operatorname{arsinh} 2}^{\operatorname{arsinh} 3} \frac{1}{\tanh x - \operatorname{sech} x} dx.$$
 [4]

END OF QUESTION PAPER

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