OCR
Oxford Cambridge and RSA

Wednesday 7 June 2017 – Morning

A2 GCE MATHEMATICS (MEI)

4757/01 Further Applications of Advanced Mathematics (FP3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4757/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **24** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Option 1: Vectors

1 Four points have coordinates A(0, 1, −4), B(5, 6, 1), C(6, 1, 4) and D(−6, 1, 1).

(i) Find the shortest distance from A to the line CD. **[5]**

(ii) Show that the shortest distance between the lines AB and CD is $\sqrt{26}$. **[5]**

(iii) Find points P and Q, lying on AB and CD respectively, such that PQ is of length $\sqrt{26}$. **[6]**

(iv) Explain why the lengths found in parts **(i)** and **(ii)** are not the same. **[1]**

(v) Find the equation of the plane Π that contains the line CD and is parallel to the line AB. **[3]**

(vi) You are given that the point E (0, −1, 1) lies on Π. Find the volume of the tetrahedron PECD. **[4]**

Option 2: Multi-variable calculus

2 A surface has equation $z = (x^2 + y^2)(x + 1)$.

(i) (*A*) Show that there is a stationary point at the origin. **[4]**

(*B*) By considering small values of x and y find the nature of this stationary point. **[3]**

(ii) (*A*) Show that there is exactly one other stationary point. Find its coordinates. **[3]**

(*B*) By considering sections of the surface at this stationary point, show that this point is neither a maximum nor a minimum. **[6]**

(iii) The point P(1, 1, 4) lies on the surface and the point Q(1 + h, 1 + h, 4 + k) is a point on the surface close to P.

Find an approximate expression for k in terms of h. **[4]**

(iv) Find the equation of the tangent plane to the surface at point P. **[4]**

Option 3: Differential geometry

3 A curve has parametric equations $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, for $0 \leqslant \theta \leqslant 2\pi$ where a is a positive constant. The point A on the curve has parameter $\theta = \frac{1}{2}\pi$.

(i) Show that

(A) the curve passes through the origin, **[1]**

(B) the arc length, s, from the origin to the point A is $2a\sqrt{2}$. **[6]**

(ii) The curve from O to A is rotated through 2π about the x-axis. Find the area of the curved surface generated. **[5]**

(iii) Find

(A) the intrinsic equation of the curve, **[4]**

(B) the centre of curvature for point A. **[8]**

Option 4: Groups

4 **(a)** The composition table for a group G of order 6 is given below.

	a	b	c	d	e	f
a	c	f	e	b	a	d
b	f	a	d	e	b	c
c	e	d	a	f	c	b
d	b	e	f	c	d	a
e	a	b	c	d	e	f
f	d	c	b	a	f	e

(i) State the identity element. **[1]**

(ii) State the order of each element. **[3]**

(iii) Write down the inverse of each element. **[3]**

(iv) Determine whether G is cyclic. **[2]**

(v) List all the proper subgroups. Comment on the order of these groups in relation to Lagrange's theorem. **[3]**

(vi) Specify an isomorphism between G and the group F consisting of $\{1,2,3,4,5,6\}$ under multiplication modulo 7. **[4]**

(b) A group H is commutative and has e as its identity element. Three elements of the group, a, b, and c have order 2, 3, and 5 respectively. The order of H is the minimum value consistent with these properties.

(i) State the order of H. **[1]**

(ii) Prove that the order of ab is 6. **[4]**

(iii) Prove that H is cyclic. **[3]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 Two bags, A and B, have a total of 3 balls in them. An event is to choose a ball at random, find the bag it is currently in, and transfer the ball to the other bag.

The transition matrix, **M**, for the number of balls in bag A is shown below.

$$\mathbf{M} = \begin{pmatrix} 0 & \frac{1}{3} & 0 & 0 \\ 1 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{2}{3} & 0 & 1 \\ 0 & 0 & \frac{1}{3} & 0 \end{pmatrix}$$

(i) Explain carefully the contents of the second column. **[3]**

(ii) Explain what is meant by a reflecting barrier in a Markov chain. Identify any reflecting barriers in this situation. **[3]**

(iii) Find $\mathbf{M}^4$ and $\mathbf{M}^5$. **[4]**

(iv) Find the limiting values of $\mathbf{M}^{2n}$ and $\mathbf{M}^{2n+1}$ as n tends to infinity. **[4]**

(v) Find the equilibrium probabilities that bag A contains 0, 1, 2 or 3 balls. Comment on your answer in relation to the results of part **(iv)**. **[4]**

(vi) Find the probability that bag A contains the same number of balls after 15 stages that it did after 12 stages. Show clearly how you obtain your answer. **[3]**

(vii) Now suppose that the situation is as before but with a total of 4 balls in the two bags. Write down the 5×5 transition matrix for bag A. **[3]**

END OF QUESTION PAPER

BLANK PAGE

BLANK PAGE