OCR
Oxford Cambridge and RSA

Wednesday 24 May 2017 – Morning

A2 GCE MATHEMATICS (MEI)

4763/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4763/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR
- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is moving, with negligible air resistance, in a complete vertical circle with centre O. When P is at its highest point the speed of P is V. The horizontal line CD lies in the plane of the motion and passes through the lowest point of the circular path of P. Fig. 1 shows the particle at a point where OP makes an angle θ with the upward vertical.

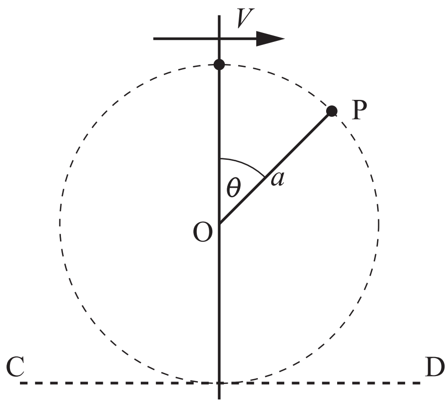

Fig. 1

(i) Show that the least possible value of V is $\sqrt{ag}$. **[2]**

(ii) Given that $V = \sqrt{ag}$, find an expression, in terms of m, g and θ, for the tension in the string when P is in the position shown in Fig. 1. **[6]**

Now consider the case $V = \sqrt{3ag}$.

(iii) Find the vertical height of P above CD when the tension in the string is equal to twice its minimum value. **[6]**

Suppose now that $V = \sqrt{kag}$, where k is a positive constant.

The string breaks if the tension in it exceeds $12mg$.

(iv) Find the set of values that k can take so that P is able to complete vertical circles. **[3]**

2 **(a)** A moving car experiences a force F due to air resistance. It is known that F depends on a product of powers of its velocity v, its cross-sectional area A and the air density ρ, and is given by

$$F = \frac{1}{2}C\rho^{\alpha}v^{\beta}A^{\gamma},$$

where C is a dimensionless constant known as the drag coefficient.

(i) Write down the dimensions of force and density. **[2]**

(ii) Use dimensional analysis to find α, β and γ. **[5]**

(b)

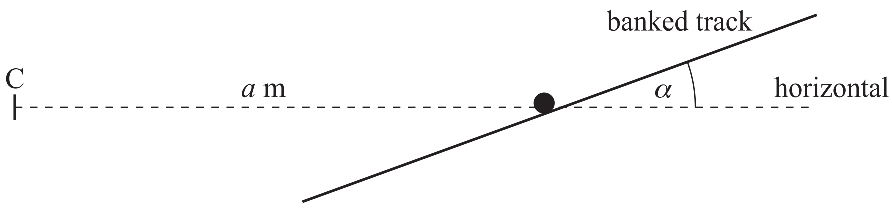

Fig. 2

A motorcyclist is riding his motorcycle around a circular banked track. The track is banked at an angle α to the horizontal, where $\tan\alpha = \frac{1}{4}$. The combined mass of the motorcycle and rider is M kg. The motion of the motorcycle and rider is modelled as a particle travelling at constant speed in a horizontal circle, with centre C and radius a m, on the banked track, as shown in Fig. 2.

(i) Given that there is no tendency for the motorcyclist to slip up or down the slope when his speed is $5\sqrt{g}$ m s^{-1}, show that $a = 100$. **[4]**

Suppose now that the coefficient of friction between the motorcyclist and the track is μ.

(ii) Given that the maximum constant speed for which motion in the horizontal circle centre C is possible is 28 m s^{-1}, find the value of μ. **[7]**

3 Fig. 3 shows a smooth plane inclined at an angle of 30° to the horizontal. A particle P of mass 3 kg lies on the plane. One end of a light elastic string, of natural length 2 m, is attached to P and the other end is fixed to a point A. One end of a second light elastic string, of natural length 1 m, is attached to P and the other end is fixed to a point B. Both strings are made from material with modulus of elasticity 12.25 N. APB is parallel to the plane on a line of greatest slope, and the distance AB is 6 m.

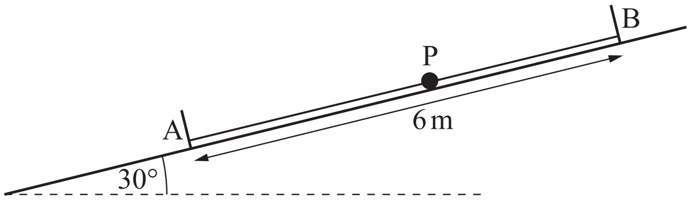

Fig. 3

The particle P moves along part of the line AB with both strings taut throughout the motion.

(i) Show that, when the extension of the string AP is x m, the tension in the string BP is $12.25(3-x)$ N. Show also that the value of x for which the system is in equilibrium is 1.2. **[4]**

The particle P is released from rest when AP = 3.35 m. At time t s, the displacement of P from its equilibrium position is y m, measured in the direction AB.

(ii) Show that the motion of P is simple harmonic with equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = -6.125y.$$

State the period of the motion. **[8]**

The point C is on the line AB, between A and B, such that AC = 3.1 m.

(iii) Find the speed of P when it is at C. **[2]**

(iv) Find the time elapsed after its release from rest until P is at C moving **up** the plane for the first time. **[5]**

4 Fig. 4.1 shows the shaded region R bounded by the curve $y = 2x^{-\frac{1}{2}}$ for $1 \leqslant x \leqslant 4$, the x-axis and the lines $x = 1$ and $x = 4$.

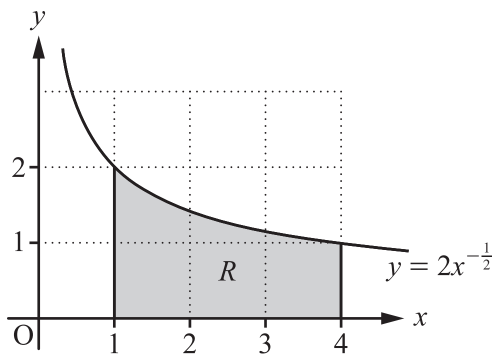

Fig. 4.1

(i) Find the exact coordinates of the centre of mass of a uniform lamina occupying the region R. **[6]**

Fig. 4.2 shows the shaded region S bounded by the curve $y = 2x^{-\frac{1}{2}}$ for $1 \leqslant x \leqslant 4$, the x-axis and the lines $x = 4$ and $y = 2x$. The line $y = 2x$ meets the curve $y = 2x^{-\frac{1}{2}}$ at the point A with coordinates (1, 2).

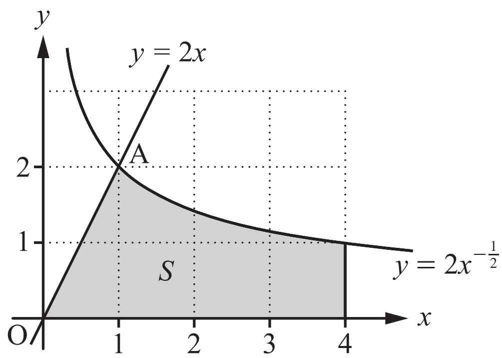

Fig. 4.2

The region S is rotated through 2π radians about the x-axis to form a uniform solid of revolution.

(ii) Show that the x-coordinate of the centre of mass of this solid is $\dfrac{39}{4(1 + 6\ln 2)}$.

(You may assume the standard results for the volume and the position of the centre of mass of a uniform solid cone.) **[8]**

(iii) The solid is suspended from a point on the circle described by A when S is rotated about the x-axis. Find the angle between AO and the vertical. **[4]**

END OF QUESTION PAPER

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