



Oxford Cambridge and RSA

**Monday 26 June 2017 – Afternoon**

**A2 GCE MATHEMATICS (MEI)**

**4764/01 Mechanics 4**

**QUESTION PAPER**



Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4764/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration: 1 hour 30 minutes**

**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ ms}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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## Section A (24 marks)

1 A car moves horizontally in a straight line with speed  $v$  at time  $t$ . The total resistance force on the car has magnitude  $kv$  where  $k$  is a positive constant. The car is powered by a rocket, which ejects burnt fuel backwards at a constant mass rate  $\lambda$  and at a constant speed  $u$  relative to the car. The initial mass of the car and the fuel is  $M$  and at time  $t$ , when some fuel still remains to be burnt, the mass of the car and the remaining fuel is  $m$ .

(i) Derive the differential equation  $m \frac{dv}{dt} + u \frac{dm}{dt} = -kv$ . [3]

(ii) Given that the initial speed of the car is zero, show that

$$v = \frac{\lambda u}{k} \left( 1 - \left( \frac{M - \lambda t}{M} \right)^{\frac{k}{\lambda}} \right),$$

and hence show that for small values of  $t$  the speed of the car is approximately  $\frac{\lambda u t}{M}$ . [9]

2 A particle of mass 3 kg moves along the  $x$ -axis by means of a driving force applied in the positive  $x$ -direction. There are no other forces acting on the particle. When the particle is  $x$  m from the origin O, its velocity is  $v \text{ ms}^{-1}$ . Initially  $v = 3$  and the particle is at O. The magnitude of the driving force is  $F$  N, where  $F = e^{0.1x} (v^2 - 1)^{\frac{1}{3}}$ .

(i) By solving a suitable equation of motion satisfied by the particle, show that  $F$  may be written as

$$F = \frac{2}{3} e^{0.1x} \sqrt{10e^{0.1x} - 1}. [9]$$

(ii) By using the work-energy principle, and without further integration, show that

$$\int_0^{10} e^{0.1x} \sqrt{10e^{0.1x} - 1} dx = k \left( (10e - 1)^{\frac{3}{2}} - 27 \right),$$

stating the exact value of the constant  $k$ . [3]



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## Section B (48 marks)

3 Fig. 3 shows a smooth wire in the form of a semi-circle with centre O and radius  $a$ . The wire is fixed in a vertical plane. The points C and D are at the ends of the wire at the same horizontal level as O. A small ring, P, of mass  $\lambda m$  can move freely on the wire. One end of a light inextensible string of length  $2a$  is attached to P. The string passes over a small smooth fixed pulley at C; a particle of mass  $\mu m$  hangs freely from its other end, vertically below C. One end of a second light inextensible string of length  $2a$  is attached to P. This string passes over a small smooth fixed pulley at D; a particle of mass  $\mu m$  hangs freely from its other end, vertically below D. The radius OP makes an angle  $2\theta$  with the downward vertical, where  $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$ , and  $\lambda$  and  $\mu$  are positive constants.

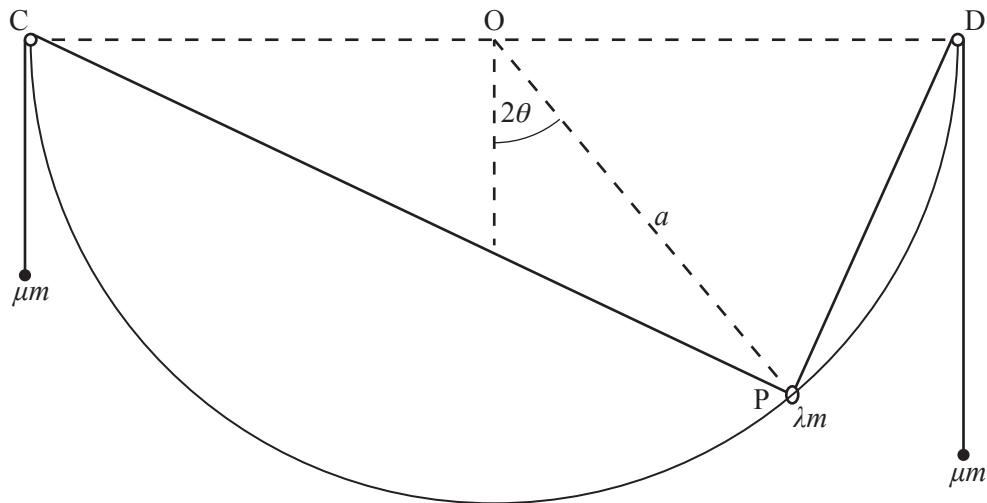


Fig. 3

(i) Find the potential energy,  $V$ , of the system relative to the level of CD, and hence show that

$$\frac{dV}{d\theta} = 2mga(\lambda \sin 2\theta - \sqrt{2}\mu \sin \theta). \quad [8]$$

(ii) Show that there are three values of  $\theta$  for which the system is in equilibrium provided that  $\lambda < \mu < \sqrt{2}\lambda$ . [5]

(iii) Given that there are three positions of equilibrium, establish whether each of these positions is stable or unstable. [8]

You are now given that  $\mu = 6$  and  $\lambda = 3\sqrt{2}$ .

(iv) Investigate the stability of the single equilibrium position of the system. [3]

4 A triangular lamina OAB of mass  $M$  kg has  $OA = OB$  and  $AB = 2a$  m.  $OX = 3a$  m, where X is the mid-point of AB. Fig. 4 shows this lamina in an  $x$ - $y$  plane with origin O and  $OX$  horizontal. The mass per unit area  $\rho$  kg m $^{-2}$  of the lamina is given by  $\rho = k\left(1 + \frac{x}{a}\right)$  where  $x$  m is the horizontal distance from O and  $k$  is a positive constant.

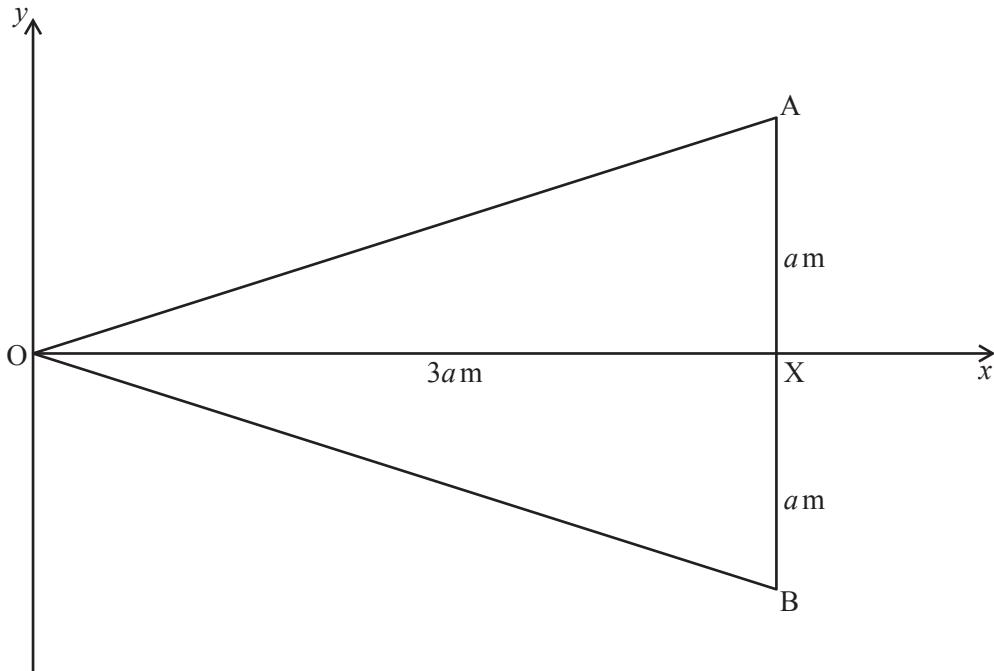


Fig. 4

(i) Show that  $M = 9ka^2$ . [5]

(ii) Show, using integration, that the moment of inertia of the lamina about an axis through O perpendicular to the plane of the lamina is  $\frac{238}{45}Ma^2$ . [You may assume the standard formula for the moment of inertia of a thin rod about an axis through its centre perpendicular to the rod.] [7]

The lamina is free to rotate in a vertical plane about a fixed smooth horizontal axis through O perpendicular to the lamina. The lamina is released from rest with  $OX$  making an angle  $\phi$  with the downward vertical. At time  $t$  s after the lamina is released,  $OX$  makes an angle  $\theta$  with the downward vertical.

(iii) Show that the angular velocity  $\dot{\theta}$  of the lamina when it has turned through an angle  $\theta$  satisfies

$$a\dot{\theta}^2 = \sigma g(\cos \theta - \cos \phi),$$

stating the exact value of the constant  $\sigma$ . [8]

You are now given that  $a = 2.25$  and that  $\phi$  is small.

(iv) Show that the motion is approximately simple harmonic, and find the approximate time when the lamina first comes instantaneously to rest. [4]

**END OF QUESTION PAPER**