

OCR

Oxford Cambridge and RSA

Monday 26 June 2017 – Afternoon

A2 GCE MATHEMATICS (MEI)

4764/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4764/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ ms}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (24 marks)

- 1 A car moves horizontally in a straight line with speed v at time t . The total resistance force on the car has magnitude kv where k is a positive constant. The car is powered by a rocket, which ejects burnt fuel backwards at a constant mass rate λ and at a constant speed u relative to the car. The initial mass of the car and the fuel is M and at time t , when some fuel still remains to be burnt, the mass of the car and the remaining fuel is m .

(i) Derive the differential equation $m \frac{dv}{dt} + u \frac{dm}{dt} = -kv$. [3]

- (ii) Given that the initial speed of the car is zero, show that

$$v = \frac{\lambda u}{k} \left(1 - \left(\frac{M - \lambda t}{M} \right)^{\frac{k}{\lambda}} \right),$$

and hence show that for small values of t the speed of the car is approximately $\frac{\lambda ut}{M}$. [9]

- 2 A particle of mass 3 kg moves along the x -axis by means of a driving force applied in the positive x -direction. There are no other forces acting on the particle. When the particle is x m from the origin O, its velocity is v m s⁻¹. Initially $v = 3$ and the particle is at O. The magnitude of the driving force is F N, where $F = e^{0.1x}(v^2 - 1)^{\frac{1}{3}}$.

- (i) By solving a suitable equation of motion satisfied by the particle, show that F may be written as

$$F = \frac{2}{3} e^{0.1x} \sqrt{10e^{0.1x} - 1}. \quad [9]$$

- (ii) By using the work-energy principle, and without further integration, show that

$$\int_0^{10} e^{0.1x} \sqrt{10e^{0.1x} - 1} dx = k \left((10e - 1)^{\frac{3}{2}} - 27 \right),$$

stating the exact value of the constant k . [3]

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Section B (48 marks)

- 3 Fig. 3 shows a smooth wire in the form of a semi-circle with centre O and radius a . The wire is fixed in a vertical plane. The points C and D are at the ends of the wire at the same horizontal level as O. A small ring, P, of mass λm can move freely on the wire. One end of a light inextensible string of length $2a$ is attached to P. The string passes over a small smooth fixed pulley at C; a particle of mass μm hangs freely from its other end, vertically below C. One end of a second light inextensible string of length $2a$ is attached to P. This string passes over a small smooth fixed pulley at D; a particle of mass μm hangs freely from its other end, vertically below D. The radius OP makes an angle 2θ with the downward vertical, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$, and λ and μ are positive constants.

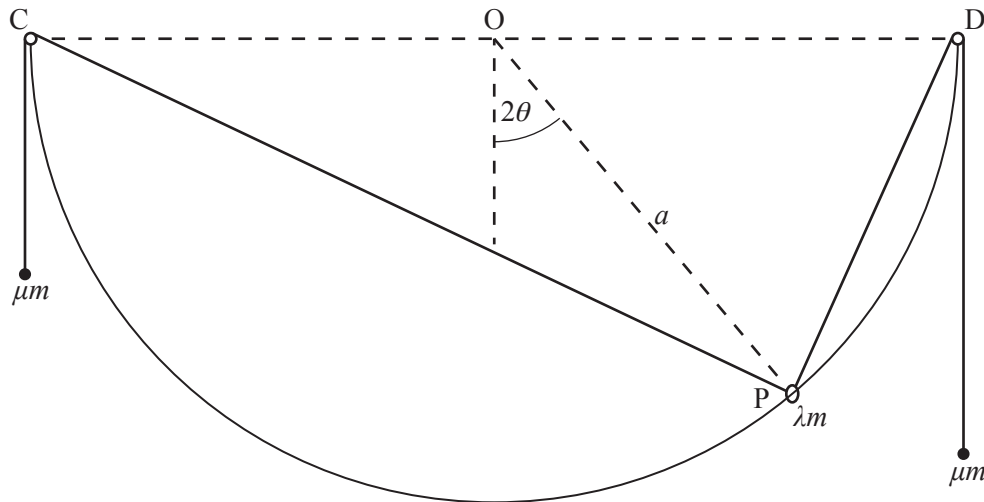


Fig. 3

- (i) Find the potential energy, V , of the system relative to the level of CD, and hence show that

$$\frac{dV}{d\theta} = 2mga(\lambda \sin 2\theta - \sqrt{2}\mu \sin \theta). \quad [8]$$

- (ii) Show that there are three values of θ for which the system is in equilibrium provided that $\lambda < \mu < \sqrt{2}\lambda$. [5]

- (iii) Given that there are three positions of equilibrium, establish whether each of these positions is stable or unstable. [8]

You are now given that $\mu = 6$ and $\lambda = 3\sqrt{2}$.

- (iv) Investigate the stability of the single equilibrium position of the system. [3]

- 4 A triangular lamina OAB of mass M kg has $OA = OB$ and $AB = 2a$ m. $OX = 3a$ m, where X is the mid-point of AB . Fig. 4 shows this lamina in an x - y plane with origin O and OX horizontal. The mass per unit area ρ kg m^{-2} of the lamina is given by $\rho = k\left(1 + \frac{x}{a}\right)$ where x m is the horizontal distance from O and k is a positive constant.

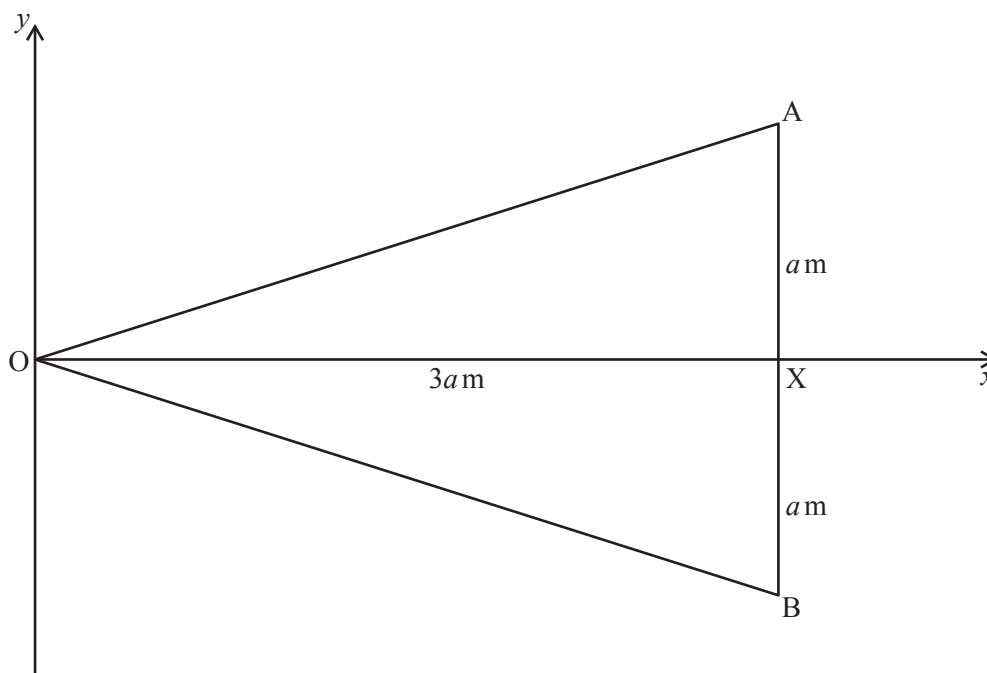


Fig. 4

- (i) Show that $M = 9ka^2$. [5]

- (ii) Show, using integration, that the moment of inertia of the lamina about an axis through O perpendicular to the plane of the lamina is $\frac{238}{45}Ma^2$. [You may assume the standard formula for the moment of inertia of a thin rod about an axis through its centre perpendicular to the rod.] [7]

The lamina is free to rotate in a vertical plane about a fixed smooth horizontal axis through O perpendicular to the lamina. The lamina is released from rest with OX making an angle ϕ with the downward vertical. At time t s after the lamina is released, OX makes an angle θ with the downward vertical.

- (iii) Show that the angular velocity $\dot{\theta}$ of the lamina when it has turned through an angle θ satisfies

$$a\dot{\theta}^2 = \sigma g(\cos \theta - \cos \phi),$$

stating the exact value of the constant σ .

[8]

You are now given that $a = 2.25$ and that ϕ is small.

- (iv) Show that the motion is approximately simple harmonic, and find the approximate time when the lamina first comes instantaneously to rest. [4]

END OF QUESTION PAPER