OCR
Oxford Cambridge and RSA

Wednesday 7 June 2017 – Morning

A2 GCE MATHEMATICS (MEI)

4772/01 Decision Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4772/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Turn over

1 **(a)** The philosopher Epimenides is said to have stated "All Cretans are liars". Epimenides was a Cretan.

Analyse the statement, starting with a consideration of what might be meant when someone is said to be a liar. **[3]**

(b) **(i)** Use a truth table to prove that "$\wedge$" is distributive over "$\vee$", i.e. that $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$. **[3]**

(ii) Use a truth table to prove that "$\vee$" is distributive over "$\wedge$". **[2]**

(c) **(i)** Explain why $(a \Rightarrow b) \Leftrightarrow (\sim a \vee b)$. **[1]**

(ii) Draw a combinatorial circuit for $(a \Rightarrow (b \vee c))$. **[3]**

(iii) Use Boolean algebra to prove that $\sim (a \wedge \sim (b \vee c)) \Leftrightarrow (a \Rightarrow (b \vee c))$. **[2]**

(iv) It is not the case that Adrian and neither Brian nor Claire were guilty. So, if Adrian was guilty, what can you deduce about Brian and Claire? State and prove. **[2]**

2 Yvette is to buy tickets for a one-way journey. She can choose between a scheduled airline, a budget airline and a charter airline. She knows the costs of the tickets and other travel costs, but she also wants to take account of the time of her journey. This varies between the airlines because they use different airports, and because the budget airline involves two connecting flights. The time is also subject to delay, which Yvette has researched. Yvette costs her time at £50 per hour.

Airline	Cost (i.e. all travel costs)	Total journey time (hours)	Probability of 1 hour delay	Probability of 2 hour delay	Probability of 24 hour delay
Scheduled	£180	3	0.05	0.01	0
Budget	£80	4.5	0.1	0.03	0.02*
Charter	£120	4	0.2	0.03	0

(*… as a consequence of missing the connecting flight)

(i) Draw a decision tree for Yvette. **[4]**

(ii) Evaluate the EMV at each node of your tree, and give the airline which minimises the EMV of the cost of Yvette's journey. **[8]**

(iii) Investigate the consequences of using a square root utility function applied to total costs. Give the airline with the best expected utility and that value. **[2]**

Yvette discovers a website which will tell her before she buys her tickets whether or not the budget airline plane for the first leg is delayed on its previous flight. If it is not delayed then the connection is assured, although other delays may take place as before.

(iv) What is the EMV of this information? **[2]**

3 **(a)** Consider applying Floyd's algorithm to a complete undirected network on 5 vertices to find the complete network of shortest routes.

(i) How many comparisons are needed during the first iteration? **[1]**

(ii) How many comparisons are needed in total? **[1]**

Now consider repeatedly applying Dijkstra's algorithm to a complete undirected network on 5 vertices. The first iteration finds all of the shortest routes from the first vertex. In the second iteration the first vertex and its arcs are deleted and all of the shortest routes are found from the second vertex … and so on.

(iii) How many comparisons are needed during the first iteration? **[2]**

(iv) How many comparisons are needed in total? **[2]**

(b) The unbracketed numbers in the table give direct distances between the vertices of a network, where direct connections exist. These are also the shortest distances. The bracketed numbers give shortest indirect distances, where no direct connections exist.

	A	B	C	D	E	F
A		32	27	(54)	41	31
B	32		38	22	45	33
C	27	38		(47)	18	(58)
D	(54)	22	(47)		29	23
E	41	45	18	29		(52)
F	31	33	(58)	23	(52)	

(i) By deleting A and its arcs, and by finding a minimum connector for the remaining vertices, construct a lower bound for the solution to the travelling salesperson problem in this network. **[4]**

(ii) Make three attempts to apply the nearest neighbour algorithm to find a good solution to the TSP. Start at vertex A for your first attempt, at vertex B for your second attempt and at vertex C for your final attempt, and give the best of your attempts. **[3]**

(iii) Use the route inspection algorithm to find an optimal solution to the Chinese postperson problem in the network. Give an optimal route. **[7]**

4 Ian the chef is constructing a recipe for a dish with four ingredients: pasta, sauce, cheese and olive oil. The fat, salt and sugar concentrations in each of his ingredients are listed in the table, all in units of grams per 100 grams. The table also shows the upper limits for concentrations which Ian does not want to exceed in constructing his recipe.

	pasta	sauce	cheese	olive oil	limits
fat	2	5	30	100	26
salt	0.05	1	2	0	1
sugar	3.5	2	0	0	2.5

The following LP is constructed to help Ian with constructing his recipe.

maximise $P = p1 + p2 + p3 + p4$
subject to $p1 + p2 + p3 + p4 \leqslant 1$
$2p1 + 5p2 + 30p3 + 100p4 \leqslant 26$
$0.05p1 + p2 + 2p3 \leqslant 1$
$3.5p1 + 2p2 \leqslant 2.5$
end

(i) Explain what p1, p2, p3 and p4 represent in this formulation. **[1]**

(ii) Set the problem up in an initial simplex tableau. **[5]**

(iii) Perform the first iteration of the Simplex algorithm. Pivot on an element in the p1 column. **[5]**

There are many solutions to the LP, including $\left(\frac{1}{3}, \frac{2}{3}, 0, 0\right)$ and $(0.405, 0.000, 0.490, 0.105)$, where the solutions are given in the form (p1, p2, p3, p4) either exact or correct to 3 decimal places.

(iv) Obtain the fat, salt and sugar concentrations for the two solutions quoted above. **[2]**

Ian adds the following constraints …

- The pasta content must be at least 40%.
- The sauce content must be at least 30%.
- There must be at least 10% cheese.
- There must be at least 5% olive oil.

(v) Extend your initial tableau from part (ii) to model this extended problem using two-phase Simplex. Do not attempt to solve the problem. **[6]**

An LP package gives a solution to this extended problem as (0.4, 0.3, 0.1, 0.2)

(vi) Interpret this solution. **[1]**

END OF QUESTION PAPER

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