OCR
Oxford Cambridge and RSA

Wednesday 28 June 2017 – Morning

A2 GCE MATHEMATICS (MEI)

4798/01 Further Pure Mathematics with Technology (FPT)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4798/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software

Duration: Up to 2 hours

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a computer algebra system, a spreadsheet, a programming language and graph-plotting software throughout the examination.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

DC (LK) 138509/3

1 This question concerns curves with parametric equations

$$x = \cos t,\ y = \sin nt,$$

where n is a positive integer and $0 \leqslant t < 2\pi$.

(i) Sketch the curves in the cases $n = 2$, $n = 3$ and $n = 4$. **[4]**

(ii) For the case $n = 4$, find the values of t where the curve crosses the x-axis. Hence show that the curve crosses itself at three points and state the x-coordinates of these points. **[4]**

(iii) For the case $n = 3$, find the coordinates of the points where the tangent to the curve is parallel to the x-axis. **[4]**

(iv) Show that, for any positive integer value of n, the curve has $2n$ distinct points where the tangent to the curve is parallel to the x-axis. **[6]**

(v) For the case $n = 2$, find a cartesian equation of the curve in the form $y^2 = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is a polynomial in x. Hence find the total area enclosed by the curve. **[7]**

2 This question concerns the function $\mathrm{f}(z) = \mathrm{e}^z$ for $z \in \mathbb{C}$.

(i) The values of $\mathrm{f}(2+\mathrm{i})$ and $\mathrm{f}(2.01+\mathrm{i})$ are denoted by z_1 and z_2 respectively. Find z_1 and z_2, giving real and imaginary parts correct to 3 decimal places.

Find the value of $\dfrac{z_2 - z_1}{0.01}$ and explain why this is approximately equal to z_1. **[5]**

(ii) Construct a spreadsheet to demonstrate that $\lim\limits_{h\to 0}\left(\dfrac{\mathrm{f}(2+\mathrm{i}+h)-\mathrm{f}(2+\mathrm{i})}{h}\right) = \mathrm{e}^{2+\mathrm{i}}$ where $h \in \mathbb{R}$.

State which values of h you have used and the expression(s) you have evaluated. Quoting sufficient values from your spreadsheet, explain how the result is demonstrated.

Use your spreadsheet to find, correct to 1 significant figure, the largest value of h such that

$$\mathrm{Re}\left(\frac{\mathrm{f}(2+\mathrm{i}+h)-\mathrm{f}(2+\mathrm{i})}{h} - \mathrm{e}^{2+\mathrm{i}}\right) < 0.01.$$ **[6]**

(iii) Show that $\mathrm{f}(k+\pi\mathrm{i})$ is a negative real number for all $k \in \mathbb{R}$. Show also that there are no values of z such that $\mathrm{f}(z) = 0$. **[6]**

(iv) Show that the points on an Argand diagram representing the roots of the equation $\mathrm{f}(z) = -2$ lie on a straight line and write down the equation of this line.

Now taking z to denote a general point on this line, find $\mathrm{f}(z)$ and hence describe the locus of points given by $\mathrm{f}(z)$ as z varies. **[7]**

3 This question investigates those positive integers, n, which can be expressed as the sum of the squares of two positive integers a and b, and those which can be expressed as the sum of the squares of three positive integers a, b and c.

(i) Show that if $n = a^2 + b^2$ then $a < \sqrt{n}$ and $b < \sqrt{n}$. **[1]**

(ii) Create a program that will find all possible values of a and b such that $n = a^2 + b^2$ for a given value of n, where $a \leqslant b$. You should write out your program in full.

Use your program to find all such ways of expressing n in the form $a^2 + b^2$ for $n = 1009$, $n = 1019$ and $n = 1037$. **[8]**

(iii) By considering all the possible values of $a^2 \pmod 4$ and $b^2 \pmod 4$, show that if $n \equiv 3 \pmod 4$ then n cannot be expressed in the form $a^2 + b^2$. **[5]**

(iv) Edit your program so that it will find all possible values of a, b and c such that $n = a^2 + b^2 + c^2$ for a given value of n, where $a \leqslant b \leqslant c$. You should state the changes you have made to your program.

Use your program to find all such ways of expressing n in the form $a^2 + b^2 + c^2$ for $n = 161$ and $n = 167$. **[4]**

(v) Show that if $n \equiv 7 \pmod 8$ then n cannot be expressed in the form $a^2 + b^2 + c^2$. **[5]**

END OF QUESTION PAPER