



Oxford Cambridge and RSA

Wednesday 28 June 2017 – Morning**A2 GCE MATHEMATICS (MEI)****4798/01** Further Pure Mathematics with Technology (FPT)**QUESTION PAPER**

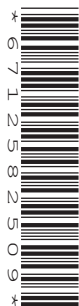
Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4798/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator
- Computer with appropriate software

Duration: Up to 2 hours**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a computer algebra system, a spreadsheet, a programming language and graph-plotting software throughout the examination.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

- 1 This question concerns curves with parametric equations

$$x = \cos t, y = \sin nt,$$

where n is a positive integer and $0 \leq t < 2\pi$.

- (i) Sketch the curves in the cases $n = 2$, $n = 3$ and $n = 4$. [4]

- (ii) For the case $n = 4$, find the values of t where the curve crosses the x -axis. Hence show that the curve crosses itself at three points and state the x -coordinates of these points. [4]

- (iii) For the case $n = 3$, find the coordinates of the points where the tangent to the curve is parallel to the x -axis. [4]

- (iv) Show that, for any positive integer value of n , the curve has $2n$ distinct points where the tangent to the curve is parallel to the x -axis. [6]

- (v) For the case $n = 2$, find a cartesian equation of the curve in the form $y^2 = f(x)$, where $f(x)$ is a polynomial in x . Hence find the total area enclosed by the curve. [7]

- 2 This question concerns the function $f(z) = e^z$ for $z \in \mathbb{C}$.

- (i) The values of $f(2+i)$ and $f(2.01+i)$ are denoted by z_1 and z_2 respectively. Find z_1 and z_2 , giving real and imaginary parts correct to 3 decimal places.

Find the value of $\frac{z_2 - z_1}{0.01}$ and explain why this is approximately equal to z_1 . [5]

- (ii) Construct a spreadsheet to demonstrate that $\lim_{h \rightarrow 0} \left(\frac{f(2+i+h) - f(2+i)}{h} \right) = e^{2+i}$ where $h \in \mathbb{R}$.

State which values of h you have used and the expression(s) you have evaluated. Quoting sufficient values from your spreadsheet, explain how the result is demonstrated.

Use your spreadsheet to find, correct to 1 significant figure, the largest value of h such that

$$\operatorname{Re} \left(\frac{f(2+i+h) - f(2+i)}{h} - e^{2+i} \right) < 0.01. \quad [6]$$

- (iii) Show that $f(k + \pi i)$ is a negative real number for all $k \in \mathbb{R}$. Show also that there are no values of z such that $f(z) = 0$. [6]

- (iv) Show that the points on an Argand diagram representing the roots of the equation $f(z) = -2$ lie on a straight line and write down the equation of this line.

Now taking z to denote a general point on this line, find $f(z)$ and hence describe the locus of points given by $f(z)$ as z varies. [7]

- 3** This question investigates those positive integers, n , which can be expressed as the sum of the squares of two positive integers a and b , and those which can be expressed as the sum of the squares of three positive integers a , b and c .

(i) Show that if $n = a^2 + b^2$ then $a < \sqrt{n}$ and $b < \sqrt{n}$. [1]

(ii) Create a program that will find all possible values of a and b such that $n = a^2 + b^2$ for a given value of n , where $a \leq b$. You should write out your program in full.

Use your program to find all such ways of expressing n in the form $a^2 + b^2$ for $n = 1009$, $n = 1019$ and $n = 1037$. [8]

(iii) By considering all the possible values of $a^2 \pmod{4}$ and $b^2 \pmod{4}$, show that if $n \equiv 3 \pmod{4}$ then n cannot be expressed in the form $a^2 + b^2$. [5]

(iv) Edit your program so that it will find all possible values of a , b and c such that $n = a^2 + b^2 + c^2$ for a given value of n , where $a \leq b \leq c$. You should state the changes you have made to your program.

Use your program to find all such ways of expressing n in the form $a^2 + b^2 + c^2$ for $n = 161$ and $n = 167$. [4]

(v) Show that if $n \equiv 7 \pmod{8}$ then n cannot be expressed in the form $a^2 + b^2 + c^2$. [5]

END OF QUESTION PAPER

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