



GCE

Mathematics (MEI)

Unit **4757**: Further Applications of Advanced Mathematics

Advanced GCE

Mark Scheme for June 2017

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the

establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

- h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Mark scheme 4757 June 2017

Question		Answer	Marks	Guidance
1	(i)	$\mathbf{c} - \mathbf{a} = \begin{pmatrix} 6 \\ 0 \\ 8 \end{pmatrix}, \quad \overline{\mathbf{CD}} = \begin{pmatrix} -12 \\ 0 \\ -3 \end{pmatrix}, \quad (\mathbf{c} - \mathbf{a}) \times \overline{\mathbf{CD}} = \begin{pmatrix} 0 \\ -78 \\ 0 \end{pmatrix}$ $\Rightarrow (\mathbf{c} - \mathbf{a}) \times \overline{\mathbf{CD}} = 78, \quad \overline{\mathbf{CD}} = \sqrt{153} = 3\sqrt{17}$ $\text{Distance} = \frac{78}{3\sqrt{17}} = \frac{26\sqrt{17}}{17}$	M1 B1 M1 B1 ft A1	Finding $\mathbf{c} - \mathbf{a}$ oe Direction CD (any multiple) Length CD Method for finding distance For length CD oe (e.g. $78/\sqrt{153}$) <i>Accept 6.31</i>
		OR $\overline{\mathbf{AQ}} = (6 + 4\lambda, 0, 8 + \lambda) \quad \mathbf{d}_2 = (4, 0, 1)$ $\overline{\mathbf{AQ}} \cdot \mathbf{d}_2 = 0 \quad (\Rightarrow \lambda = -32/17)$ $\overline{\mathbf{AQ}} = (26/17)(-1, 0, 4) \quad \mathbf{AQ} = 26\sqrt{17}/17$		M1 B1 M1 A1 A1
			[5]	
	(ii)	$\mathbf{d}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{d}_2 = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{d} = \mathbf{d}_1 \times \mathbf{d}_2 = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}$ $\text{For e.g. } \mathbf{c} - \mathbf{a} = \begin{pmatrix} 6 \\ 0 \\ 8 \end{pmatrix}$ $\text{Distance} = \frac{ (\mathbf{c} - \mathbf{a}) \cdot \mathbf{d} }{ \mathbf{d} } = \frac{26}{\sqrt{26}} = \sqrt{26} \quad \mathbf{AG}$	M1 A1 M1 A1 ft A1	Vector product of directions Vector product correct Method for finding shortest distance Appropriate use of $\mathbf{c} - \mathbf{a}$ oe
			[5]	

Question			Answer	Marks	Guidance
1	(iii)		Any point on AB is $P(\lambda, 1 + \lambda, -4 + \lambda)$ Any point on CD is $Q(6 + 4\mu, 1, 4 + \mu)$ $\overrightarrow{QP} = \begin{pmatrix} \lambda - 6 - 4\mu \\ \lambda \\ -8 + \lambda - \mu \end{pmatrix}$ Perpendicular to both lines gives 2 equations: $3\lambda - 5\mu = 14$ $5\lambda - 17\mu = 32$ Solve simultaneously: $\lambda = 3, \mu = -1$ $\Rightarrow P(3, 4, -1), Q(2, 1, 3)$	B1	Both points
				B1	
				M1	Dot product on at least one
				M1	OR $QP = (\pm) (1, 3, -4)$ Solving to obtain λ and μ
				A1 A1	[OR M2 for $QP^2 = 26$ and valid method for solving] For both
		OR	Shortest distance from P to CD is $ (\lambda - 6, \lambda, \lambda - 8) \times (4, 0, 1) / \sqrt{17} = \sqrt{26}$ $\lambda^2 + (3\lambda - 26)^2 + (-4\lambda)^2 = 442; \lambda = 3$ P is $(3, 4, -1)$ Similarly for $Q(2, 1, 3)$		M1 A1 A1 M1A1A1
				[6]	
	(iv)		Distance PQ is the shortest, so the distance from A to CD will be longer.	B1	
				[1]	

Question			Answer	Marks	Guidance
1	(v)		Eqn of plane has PQ as its normal:	M1	
			$\overrightarrow{QP} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} \Rightarrow x + 3y - 4z = k$	A1	
			Substitute any point: $\Rightarrow x + 3y - 4z = -7$	A1	
				[3]	
	(vi)		$ \mathbf{EC} \times \mathbf{ED} = \left \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix} \right = \left \begin{pmatrix} -6 \\ -18 \\ 24 \end{pmatrix} \right $	M1	Vector product of two sides
			$\Rightarrow \text{Area triangle CED} = \frac{1}{2} \sqrt{6^2 + 18^2 + 24^2} = 3\sqrt{26}$	A1	Vector product correct (ft from wrong P)
			$\Rightarrow \text{Volume PCED} = \frac{1}{3} \text{Area triangle CED} \times \text{PQ}$	M1	Method for finding volume e.g. $\mathbf{EP} \cdot (\mathbf{EC} \times \mathbf{ED}) / 6$
			$\Rightarrow V = \frac{1}{3} 3\sqrt{26} \sqrt{26} = 26$	A1 cao	Allow 26 obtained from wrong P
				[4]	

Question			Answer	Mark	Guidance
2	(i)	(A)	$\frac{\partial z}{\partial x} = 3x^2 + 2x + y^2$	M1	Diffn both
			$\frac{\partial z}{\partial y} = 2y + 2xy \quad [= 2y(x+1)]$	A1	Both correct
			When $x = y = 0$, $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = 0$	A1	Allow from wrong partial derivatives
			and show that $z = 0$	B1	
				[4]	
	(i)	(B)	x^2 and $y^2 > 0$	B1	Both
			and for small $x, 1+x > 0$.	B1	Adequate explanation
			So for all small values of x and $y, z > 0$	B1	Dependent on B2
			So minimum.	[3]	

Question			Answer	Mark	Guidance
2	(ii)	(A)	<p>Either $y = 0$ and $3x^2 + 2x = 0$</p> <p>$x = 0, -\frac{2}{3}$ so one further stationary point is $\left(-\frac{2}{3}, 0, \frac{4}{27}\right)$</p> <p>Or $x = -1 \Rightarrow 3 - 2 + y^2 = 0$: No roots</p> <p>So there are no other stationary points</p>	<p>M1</p> <p>A1</p> <p>A1</p>	
				[3]	
	(ii)	(B)	<p>Taking $x = -\frac{2}{3} \Rightarrow z = \frac{1}{3}y^2 + \frac{4}{27}$</p> <p>This section has a minimum at $y = 0$</p> <p>Taking $y = 0 \Rightarrow z = x^2(x+1)$</p> <p>This section has a maximum at $x = -2/3$</p> <p>\Rightarrow Neither maximum nor minimum</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p>	<p>Correct sketch; or correct work with derivatives;</p> <p>or $x = -2/3 + h \Rightarrow z = 4/27 - h^2(1-h)$</p> <p>Final A1 is dependent on all previous marks</p> <p><i>Numerical work earns no credit unless it relates to sections of the surface</i></p>
				[6]	

Question			Answer	Marks	Guidance
2	(iii)		At P, $\frac{\partial z}{\partial x} = 6, \frac{\partial z}{\partial y} = 4$ $\Rightarrow \delta z = \frac{\partial z}{\partial x} \delta x + \frac{\partial z}{\partial y} \delta y$ $\Rightarrow k \approx 6h + 2h = 10h$	B1 ft B1 ft M1 A1	Allow evaluation at Q instead of P
		OR	$4 + k = ((1+h)^2 + (1+h)^2)(1+h+1)$ $k = 10h + 8h^2 + 2h^3$		M1 A3, 2, 1
				[4]	
	(iv)		At (1,1,4), $\frac{\partial z}{\partial x} = 6, \frac{\partial z}{\partial y} = 4$ Equation of tangent is $\begin{pmatrix} x-1 \\ y-1 \\ z-4 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix} = 0$ $\Rightarrow 6x + 4y - z = 6$	B1 ft M1 A1 ft A1 cao	Use of (6, 4, -1) M0 for (6, 4, 1) or $6x + 4y - z = c$
				[4]	

Question			Answer	Mark	Guidance
3	(iii)	(A)	$\frac{dx}{d\theta} = a(1 + \cos \theta), \quad \frac{dy}{d\theta} = a \sin \theta$ $\Rightarrow \frac{dy}{dx} = \frac{a \sin \theta}{a(1 + \cos \theta)} = \frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}} = \tan \frac{\theta}{2}$ $\Rightarrow \psi = \frac{\theta}{2} \quad \Rightarrow s = 4a \sin \frac{\theta}{2} = 4a \sin \psi$	M1 M1 A1 A1	Finding $\frac{dy}{dx}$ Use of double angles
				[4]	
	(iii)	(B)	$\rho = \frac{ds}{d\psi} = 4a \cos \psi$ $\text{At A, } \theta = \frac{\pi}{2}, \psi = \frac{\pi}{4}$ $\Rightarrow \rho = 4a \cos \frac{\pi}{4} = \frac{4a}{\sqrt{2}} = 2a\sqrt{2}$ $x = a\left(1 + \frac{\pi}{2}\right), y = a$ $\hat{\mathbf{n}} = \begin{pmatrix} -\sin \psi \\ \cos \psi \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $\Rightarrow \text{Centre has coordinates } \left(a\left(1 + \frac{\pi}{2}\right) - \frac{a2\sqrt{2}}{\sqrt{2}}, a + \frac{a2\sqrt{2}}{\sqrt{2}} \right)$ $\text{i.e. } \left(a\left(\frac{\pi}{2} - 1\right), 3a \right)$	M1 A1 A1 B1 M1 M1 A1	Differentiate intrinsic eqn; or use other formula for ρ For coordinates of A To obtain normal vector (<i>not necessarily unit vector</i>) Method for finding centre of curvature
				[8]	

Question			Answer	Mark	Guidance																								
4	(a)	(i)	Identity element is e	B1																									
				[1]																									
	(a)	(ii)	<table border="1"> <tr> <td>a</td><td>b</td><td>c</td><td>d</td><td>e</td><td>f</td></tr> <tr> <td>3</td><td>6</td><td>3</td><td>6</td><td>1</td><td>2</td></tr> </table>	a	b	c	d	e	f	3	6	3	6	1	2	B3	All correct. B2 one error B1 2 errors												
a	b	c	d	e	f																								
3	6	3	6	1	2																								
				[3]																									
	(a)	(iii)	<table border="1"> <tr> <td>a</td><td>b</td><td>c</td><td>d</td><td>e</td><td>f</td></tr> <tr> <td>c</td><td>d</td><td>a</td><td>b</td><td>e</td><td>f</td></tr> </table>	a	b	c	d	e	f	c	d	a	b	e	f	B3	All correct. B2 one error B1 2 errors												
a	b	c	d	e	f																								
c	d	a	b	e	f																								
				[3]																									
	(a)	(iv)	Yes, the group is cyclic. For this to be so at least one element has to have an order the same as the group. This so for two elements (b and d) as they have order 6 and therefore generate the set.	B1	Do not award this mark without an attempt at a reason																								
				B1	Correct reason																								
				[2]																									
	(a)	(v)	$\{e, a, c\}$ $\{e, f\}$ Order of these subgroups are 2 and 3. The order of the group is 6. 2 and 3 are factors of 6.	B1 B1 B1																									
				[3]																									
	(a)	(vi)	<table border="1"> <tr> <td>a</td><td>b</td><td>c</td><td>d</td><td>e</td><td>f</td></tr> <tr> <td>2</td><td>3</td><td>4</td><td>5</td><td>1</td><td>6</td></tr> </table> OR <table border="1"> <tr> <td>a</td><td>b</td><td>c</td><td>d</td><td>e</td><td>f</td></tr> <tr> <td>4</td><td>5</td><td>2</td><td>3</td><td>1</td><td>6</td></tr> </table>	a	b	c	d	e	f	2	3	4	5	1	6	a	b	c	d	e	f	4	5	2	3	1	6	B4	B2 one pair interchanged
a	b	c	d	e	f																								
2	3	4	5	1	6																								
a	b	c	d	e	f																								
4	5	2	3	1	6																								
				[4]																									

Question			Answer	Mark	Guidance
4	(b)	(i)	Order is 30.	B1	
				[1]	
	(b)	(ii)	$(ab)^6 = a^6b^6 = e$ because H is commutative $ab \neq e$ so not of order 1 and must be a factor of 6 $(ab)^2 = a^2b^2 = b^2 \neq e$ so not of order 2 $(ab)^3 = a^3b^3 = a \neq e$ so not of order 3 So order must be 6 AG	M1 M1 A1 A1	Using $(ab)^n = a^n b^n$ Considering all powers relevant to their argument All relevant powers correctly evaluated
				[4]	
	(b)	(iii)	Consider abc abc has order 30 So at least one element has order the same as the group and so the group is cyclic. AG	M1 A1 A1	
				[3]	

Question		Answer	Mark	Guidance
5	(i)	If there is one ball in A then it has a chance of $\frac{1}{3}$ of being chosen.	B1	Explaining the 1/3
		If it is then there will be no balls in A.		
		If one of the balls in B is chosen, with probability $\frac{2}{3}$	B1	Explaining the 2/3
		Then the number of balls in A will increase by 1 to 2. There is no chance of there being 1 ball still in bag A and no chance of there being 3.	B1	Explaining the zeros
			[3]	
	(ii)	Any reasonable definition	B1	
		0 balls is a reflecting barrier	B1	
		3 balls is a reflecting barrier	B1	
			[3]	
	(iii)	$\mathbf{M}^4 = \begin{pmatrix} .259 & 0 & .247 & 0 \\ 0 & .753 & 0 & .741 \\ .741 & 0 & .753 & 0 \\ 0 & .247 & 0 & .259 \end{pmatrix}, \quad \mathbf{M}^5 = \begin{pmatrix} 0 & .251 & 0 & .247 \\ .753 & 0 & .749 & 0 \\ 0 & .749 & 0 & .753 \\ .247 & 0 & .251 & 0 \end{pmatrix}$	B2	For \mathbf{M}^4 , B1 one error.
			B2	For \mathbf{M}^5 , B1 one error.
			[4]	
	(iv)	$\mathbf{M}^{2n} \rightarrow \begin{pmatrix} .25 & 0 & .25 & 0 \\ 0 & .75 & 0 & .75 \\ .75 & 0 & .75 & 0 \\ 0 & .25 & 0 & .25 \end{pmatrix}, \quad \mathbf{M}^{2n+1} \rightarrow \begin{pmatrix} 0 & .25 & 0 & .25 \\ .75 & 0 & .75 & 0 \\ 0 & .75 & 0 & .75 \\ .25 & 0 & .25 & 0 \end{pmatrix}$	B2	For \mathbf{M}^{2n} B1 one error
			B2	For \mathbf{M}^{2n+1} B1 one error
			[4]	

Question			Answer	Mark	Guidance
5	(v)		$\begin{pmatrix} 0 & \frac{1}{3} & 0 & 0 \\ 1 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{2}{3} & 0 & 1 \\ 0 & 0 & \frac{1}{3} & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$ $\Rightarrow \frac{1}{3}b = a, a + \frac{2}{3}c = b, \frac{2}{3}b + d = c, \frac{1}{3}c = d$ $\Rightarrow b = 3a, c = 3a, d = a$ $\Rightarrow a = 0.125, b = 0.375, c = 0.375, d = 0.125$ $a = \frac{0.25+0}{2} = 0.125 \text{ etc.}$	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p>	
				[4]	
	(vi)		<p>Considering diagonal elements of \mathbf{M}^3</p> <p>Diagonal elements of \mathbf{M}^3 are all zero</p> <p>Hence the probability is 0</p>	<p>M1</p> <p>A1</p> <p>A1</p>	
				[3]	
	(vii)		$\begin{pmatrix} 0 & 0.25 & 0 & 0 & 0 \\ 1 & 0 & 0.5 & 0 & 0 \\ 0 & 0.75 & 0 & 0.75 & 0 \\ 0 & 0 & 0.5 & 0 & 1 \\ 0 & 0 & 0 & 0.25 & 0 \end{pmatrix}$	<p>B1</p> <p>B1</p> <p>B1</p>	<p>1st and 5th columns</p> <p>2nd and 4th columns</p> <p>3rd column</p>
				[3]	

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning

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