

OXFORD CAMBRIDGE AND RSA EXAMINATIONS
A2 GCE
4753/01
MATHEMATICS (MEI)
Methods for Advanced Mathematics (C3)
QUESTION PAPER

TUESDAY 20 JUNE 2017: Afternoon
DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The centre may enlarge the Printed Answer Book.

OCR SUPPLIED MATERIALS:

None

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF



INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided. Please write clearly and in capital letters.

If you use the Printed Answer Book, write your answer to each question in the space provided or the paper provided. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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SECTION A (36 marks)

1 Differentiate $\frac{1}{(5 - 2x^3)^2}$. [3]

2 The function $f(x)$ is defined by $f(x) = |x|$, for $-1 \leq x \leq 1$.

Sketch the graph of $y = g(x)$, where $g(x) = 2 - 2f(x)$. [3]

3 Functions $f(x)$ and $g(x)$, each defined for $-1 < x < 1$, are given by $f(x) = \ln(1 - x)$ and $g(x) = x^2$.

(i) Find $f^{-1}(x)$ and state its domain and range. [4]

(ii) Show that $f(x) + f(-x) = fg(x)$. [3]

4 A curve has equation $3x^{\frac{2}{3}} + 2y^{\frac{1}{3}} = 7$.

(i) By differentiating implicitly, find $\frac{dy}{dx}$ in terms of x and y . [3]

(ii) Hence find the gradient of the curve at the point with coordinates $(1, 8)$. [2]

- 5 A liquid is being heated. At time t minutes after heating starts, its temperature, $\theta^\circ\text{C}$, is modelled by the equation**

$$\theta = 10.5 + 69.5(1 - e^{-kt}),$$

where k is a positive constant. The boiling point of the liquid is the value approached by θ as t tends to infinity.

- (i) Write down the initial temperature and the boiling point of the liquid. [2]**
 - (ii) After being heated for one minute, the liquid has a temperature of 30°C . Find k . [3]**
 - (iii) Find how long it takes from the start of the heating until the temperature is within 1°C of the boiling point. Give your answer to the nearest minute. [3]**
- 6 You are given that the sum of the interior angles of a polygon with n sides is $180(n - 2)^\circ$. Using this result, or otherwise, prove that the interior angle of a regular polygon cannot be 155° . [3]**

7 The equation of a curve is $y = \arcsin \frac{1}{2}x$.

(i) Express each of x and $\frac{dx}{dy}$ in terms of y . [2]

(ii) A point is moving on the curve, and has coordinates (x, y) at time t . When $x = 1$, the value of $\frac{dx}{dt}$ is 2.

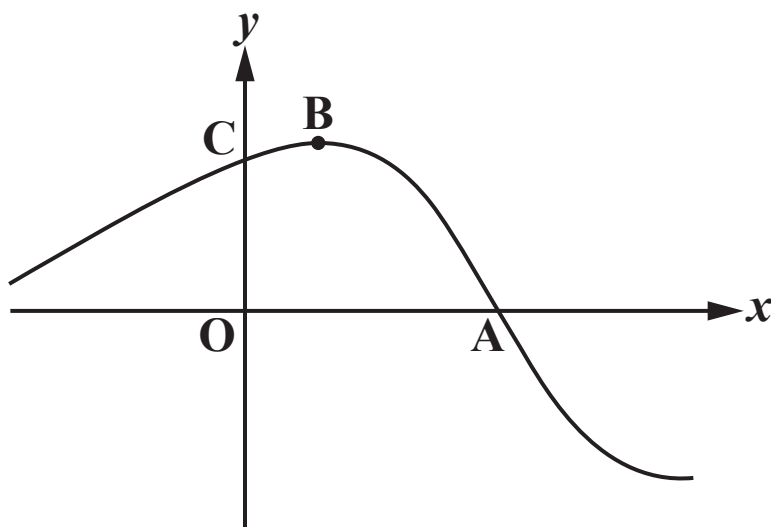
Find the exact value of $\frac{dy}{dt}$ at this instant. [5]

SECTION B (36 marks)

8 Fig. 8 shows part of the curve $y = \frac{\cos x}{2 - \sin x}$.

The curve intersects the x - and y -axes at A and C respectively, and has a turning point at B.

FIG. 8



- (i) Write down the coordinates of A and C. [2]
- (ii) Find $\frac{dx}{dy}$ and the exact coordinates of B. [7]
- (iii) (A) Using integration by substitution, or otherwise, find the exact area of the region enclosed by the curve, the y -axis and the positive x -axis. [4]

(B) The line $x = k$ divides this region into two parts of equal area. Show that $k = \arcsin(2 - \sqrt{2})$. [5]

9 A curve has equation $y = f(x)$, where $f(x) = x^3 e^{-x^2}$.

- (i) Show that $f(x)$ is an odd function, and interpret this result in terms of the graph of the curve $y = f(x)$. [3]**
- (ii) Find the coordinates of the stationary points of the curve. Give answers correct to 2 decimal places where appropriate. [7]**
- (iii) Sketch the curve for $-2 \leq x \leq 2$. [2]**
- (iv) (A) Show, using the substitution $t = x^2$, that $\int f(x) dx$ may be expressed as $\int kt e^{-t} dt$, where k is a constant to be determined. [2]**
- (B) Hence find the exact area of the region enclosed by the curve $y = f(x)$, the positive x -axis and the line $x = 2$. [4]**

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