

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4754/01A

MATHEMATICS (MEI)

**Applications of Advanced Mathematics
(C4) Paper A**

QUESTION PAPER

FRIDAY 23 JUNE 2017: Morning

**DURATION: 1 hour 30 minutes
plus your additional time allowance**

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

None

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF



INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

If you use the Printed Answer Book, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

**This paper will be followed by PAPER B:
COMPREHENSION.**

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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SECTION A (36 marks)

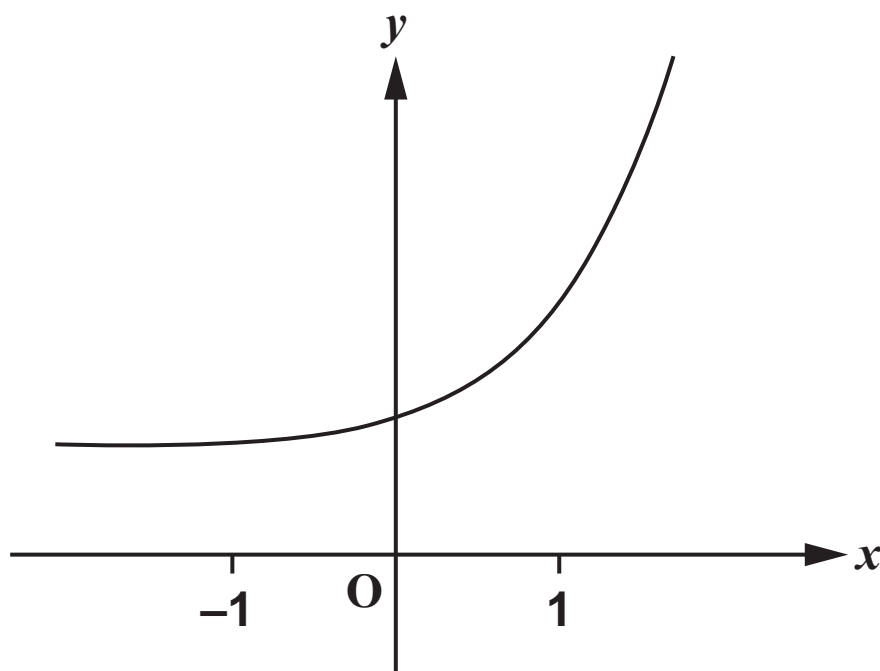
- 1 (i) Express $\frac{5-x}{(2-x)(1+x)}$ in partial fractions. [3]
- (ii) Hence or otherwise find the first 3 terms of the binomial expansion of $\frac{5-x}{(2-x)(1+x)}$ in ascending powers of x . [5]

- 2 The equation of a line is $\mathbf{r} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and the equation of a plane is $3x + 4y - z = 17$.

- (i) Find the coordinates of the point of intersection of the line and the plane. [4]
- (ii) Find the acute angle between the line and the normal to the plane. [4]

- 3 Fig. 3 shows the curve $y = \sqrt{1 + e^{2x}}$.

FIG. 3



The value of $\int_{-1}^1 \sqrt{1 + e^{2x}} dx$ is to be estimated using the trapezium rule. T_2 and T_4 are the estimates obtained from the trapezium rule using 2 strips and 4 strips respectively.

- (i) Explain whether T_4 is greater or less than T_2 . [2]
- (ii) Evaluate T_4 , giving your answer to 3 significant figures. [4]
- 4 Vectors \mathbf{u} and \mathbf{v} are given by $\mathbf{u} = \mathbf{i} - 7\mathbf{j} - 2\mathbf{k}$ and $\mathbf{v} = a\mathbf{i} + b\mathbf{j} + 5\mathbf{k}$, where a and b are constants.
- Find a and b given that the magnitude of \mathbf{v} is $\sqrt{27}$ and that \mathbf{u} and \mathbf{v} are perpendicular. [6]

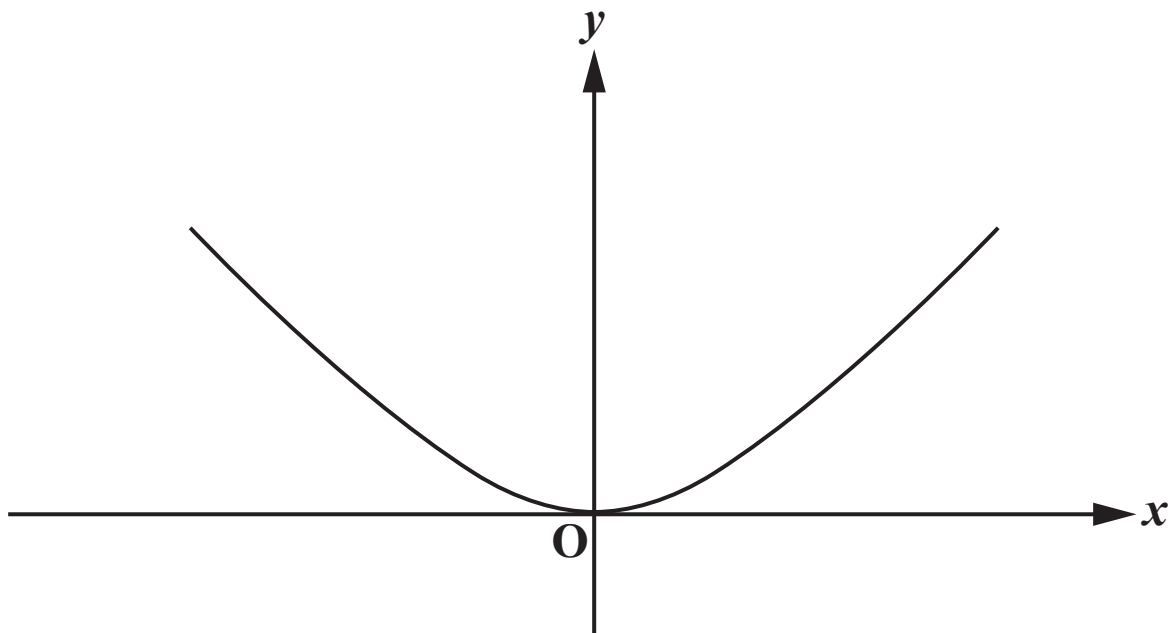
- 5 Solve the equation $4 \tan \theta \tan 2\theta = 1$, for $0^\circ < \theta < 180^\circ$.
[4]
- 6 The number of bacteria in a population at time t is denoted by P . The rate of increase of P is proportional to the square root of P .
- (i) Write down a differential equation relating P , the time t , and a constant of proportionality k . [1]
- (ii) Verify that $P = (A + Bt)^2$, where A and B are constants, satisfies the differential equation, and find k in terms of B . [3]

SECTION B (36 marks)

- 7 The curve shown in Fig. 7 passes through the origin and satisfies the differential equation

$$\frac{dy}{dx} = \frac{9x}{4(y+3)}.$$

FIG. 7



- (i) Show by integration that the equation of the curve is $9x^2 - 4y^2 - 24y = 0$. [5]

The finite region bounded by the curve and the line $y = 2$ is rotated through 180° about the y -axis.

- (ii) Find the volume of the solid of revolution generated, giving your answer as an exact multiple of π . [4]

(iii) Use the substitutions $x = 2 \tan \theta$ and $y = 3(\sec \theta - 1)$

(A) to verify that $9x^2 - 4y^2 - 24y = 0$,

(B) to show that $\frac{9x}{4(y+3)}$ can be expressed as $k \sin \theta$, where k is a constant to be found.

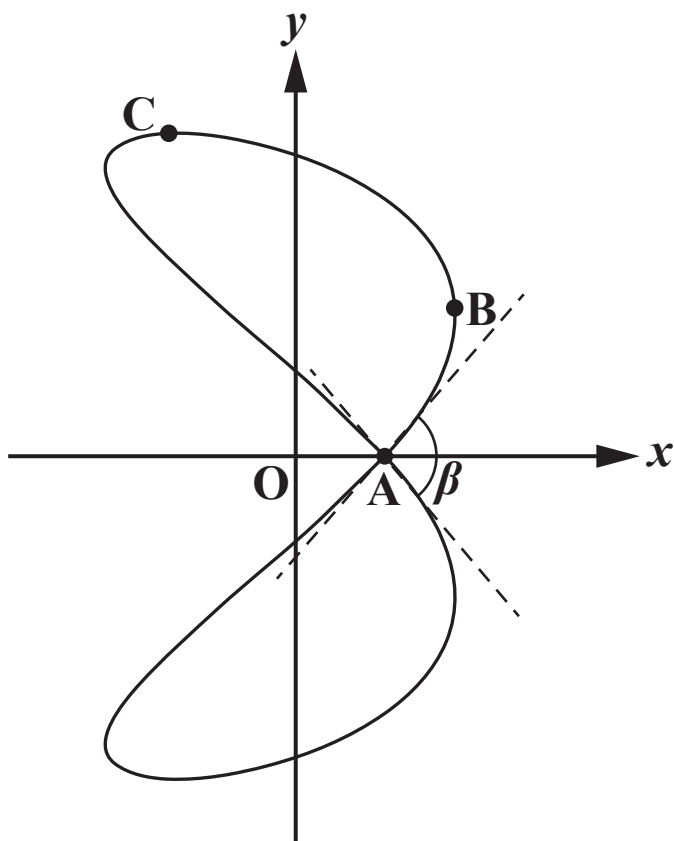
Hence find the exact gradient of the curve at the point with x -coordinate 2. [9]

8 Fig. 8 shows the curve with parametric equations

$$x = \cos 2\theta, \quad y = \cos \theta + 2 \sin \theta, \quad \text{for } -\pi < \theta \leq \pi.$$

The curve intersects the x -axis at A, and the points B and C have maximum x - and y -coordinates respectively.

FIG. 8



- (i) Find the value of θ corresponding to the point B. Hence find the coordinates of the point B. [3]**
- (ii) Express $\cos \theta + 2 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. [5]**
- (iii) Hence find the coordinates of the points A and C. [5]**

The angle β is the angle between the tangents to the curve at A.

- (iv) Find $\frac{dy}{dx}$ in terms of θ . Hence, assuming the scales of the x - and y -axes are equal, find β , giving the answer in radians correct to 2 decimal places. [You may assume the curve is symmetrical about the x -axis.] [5]

END OF QUESTION PAPER

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