OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4756/01

MATHEMATICS (MEI)

Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

MONDAY 26 JUNE 2017: Afternoon

DURATION: 1 hour 30 minutes plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

None

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (KN) 149798

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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SECTION A (54 marks)

1 **(a)** **(i)** By differentiating the equation $a\tan y = x$ show that

$$\int \frac{1}{a^2+x^2}\,\mathrm{d}x = \frac{1}{a}\arctan\left(\frac{x}{a}\right)+c\ . \quad [3]$$

The cartesian equation of an ellipse is $\frac{x^2}{4}+\frac{y^2}{9}=1$.

(ii) Show that the polar equation of the ellipse may be written in the form

$$r^2 = \frac{36\sec^2\theta}{9+4\tan^2\theta}\ . \quad [3]$$

(iii) By using the substitution $3u = 2\tan\theta$ show that the area enclosed by the ellipse and the lines $\theta = 0$ and $\theta = \frac{\pi}{4}$ is $3\arctan\left(\frac{2}{3}\right)$. [7]

(b) Obtain the first three terms of the Maclaurin series for $\mathrm{f}(x)$, where $\mathrm{f}(x) = \arctan(1+x)$. [5]

2 **(a)** The infinite series C and S are defined as follows.

$$C = -\frac{1}{2}\cos\theta + \frac{1}{4}\cos 2\theta - \frac{1}{8}\cos 3\theta + \ldots$$

$$S = -\frac{1}{2}\sin\theta + \frac{1}{4}\sin 2\theta - \frac{1}{8}\sin 3\theta + \ldots$$

By considering $C + \mathrm{j}S$, show that

$$S = \frac{-2\sin\theta}{5 + 4\cos\theta}.$$

Find a corresponding expression for C. [9]

(b) In an Argand diagram, O is the origin and points A and B are represented by the complex conjugate pair z_1 and z_2 respectively, where $0 < \arg z_1 < \frac{\pi}{2}$. The triangle OAB has side OA of length a.

(i) Show the above information on an Argand diagram. [1]

(ii) Show that $z_1 z_2$ is real, giving its value in terms of a. [2]

Triangle OAB is rotated anti-clockwise about the origin through γ radians, where $0 < \gamma < 2\pi$, and then enlarged through the origin with scale factor 3. The resulting new positions of A and B are represented by the complex numbers z_3 and z_4 respectively, where z_3 and z_4 form another complex conjugate pair.

(iii) State the value of γ. [1]

(iv) Find, in polar form (modulus-argument form), the complex number $\dfrac{z_3}{z_1}$. [2]

(v) Given that, in the original triangle OAB, AB also has length a, find the complex number $\dfrac{z_1}{z_4}$, giving your answer in the form $x + \mathrm{j}y$, where x and y are exact real numbers. [3]

3 (a) You are given the matrix $\mathbf{M} = \begin{pmatrix} k & 2 & 1 \\ 3 & -1 & 2 \\ 1 & 2 & -2 \end{pmatrix}$.

(i) Find the value of k for which $\mathbf{M}$ does not have an inverse. [3]

(ii) Find $\mathbf{M}^{-1}$ in terms of k. [4]

(b) The matrix $\mathbf{Q}$ is given by $\mathbf{Q} = \begin{pmatrix} 3 & 3 \\ 4 & 7 \end{pmatrix}$.

(i) Find the eigenvalues and corresponding eigenvectors of $\mathbf{Q}$. [5]

(ii) State a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{Q} = \mathbf{PDP}^{-1}$. [2]

(iii) Show that, for $n \geqslant 1$, $\mathbf{Q}^n = \dfrac{1}{8}\begin{pmatrix} 6 + 2\varphi & 3\varphi - 3 \\ 4\varphi - 4 & 6\varphi + 2 \end{pmatrix}$, where $\varphi = 9^n$. [4]

SECTION B (18 marks)

4 (i) Prove, from definitions involving exponentials, that $\operatorname{sech}^2 x + \tanh^2 x = 1$. [4]

(ii) Prove that

$$\operatorname{artanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right).$$

State the set of values of x for which this is valid. [5]

(iii) Solve the equation

$$3(\tanh^2 x - \operatorname{sech}^2 x) = \tanh x - 2\ ,$$

giving your answers in an exact logarithmic form. [5]

(iv) Find the exact value of

$$\int_{\operatorname{arsinh} 2}^{\operatorname{arsinh} 3} \frac{1}{\tanh x - \operatorname{sech} x}\,\mathrm{d}x\ .\ [4]$$

END OF QUESTION PAPER