OXFORD CAMBRIDGE AND RSA EXAMINATIONS

AS GCE

4776/01

MATHEMATICS (MEI)

Numerical Methods

QUESTION PAPER

TUESDAY 20 JUNE 2017:
Afternoon

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED 24pt

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:
None

OTHER MATERIALS REQUIRED:
Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (LK) 152378

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INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or the paper provided by the centre. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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SECTION A (36 marks)

1 (i) Calculate the relative error in using the approximation $\pi \approx \frac{355}{113}$. [2]

The diameter of a circle is 226.3 cm. It is decided to use the approximation $\pi \approx \frac{355}{113}$ and to round the diameter to the nearest whole number. Find the magnitude of the relative error in using these approximations to calculate

(ii) the circumference of the circle, [2]

(iii) the area of the circle. [2]

2 The following data were collected during an experiment.

x	1	2	4
y	0	4.5	29.7

(i) State, with a reason, whether or not it is possible to use Newton's forward difference interpolation formula to construct a polynomial of degree 2 for these data. [1]

(ii) Use a suitable method to construct an interpolating polynomial of degree 2 for this data. Give your answer in the form $ax^2 + bx + c$. [5]

(iii) Use your answer to part (ii) to estimate the value of y when $x = 2.5$. [2]

3 **(i)** Show that the equation $x^5 - 3x - 1 = 0$ has a root α, such that $-1 < \alpha < 0$. [2]

(ii) Use the iteration

$$x_{r+1} = \frac{x_r^5 - 1}{3},$$

to find α correct to five decimal places. Start at $x = 0$. [3]

You are given that there is another root to the equation, β, such that $\beta \approx 1.39$.

(iii) Determine whether the iteration in (ii) may be used to find β to greater accuracy. [3]

4 The formula $I = \dfrac{(Z_2 + Z_1)^2}{(Z_2 - Z_1)^2}$ occurs in physics.

You are given that $Z_1 = 21.1$ and $Z_2 = 20.9$, correct to one decimal place.

(i) Calculate the range of possible values of I. [4]

(ii) Explain why this range is so large. [2]

5 The function $f(x)$ has the values shown in the table.

x	0.3	0.35	0.375	0.425	0.45	0.5
$f(x)$	0.897 31	0.879 32	0.871 16	0.856 80	0.850 70	0.840 90

(i) Explain why it is not possible to use the forward difference method to estimate $f'(0.4)$ using these data. [1]

(ii) Calculate three estimates of $f'(0.4)$ using the central difference method. Give your answers correct to 4 decimal places. Hence state the value of $f'(0.4)$ to the accuracy which seems justified. [5]

(iii) Given that $f(0.4) = 0.863\,63$, find an approximation to $f(0.41)$. [2]

SECTION B (36 marks)

6 (i) By calculating $0.1^{(0.1^3)}$, $0.01^{(0.01^3)}$ and $0.001^{(0.001^3)}$ show that $x^{(x^3)}$ tends to 1 as x tends to 0. [2]

(ii) Identify the difficulty with using the trapezium rule to evaluate $\int_0^1 x^{(x^3)}\,dx$.

Use your answers to part (i) to suggest a way of addressing this difficulty. [2]

(iii) Use the mid-point rule with $h = 1$, $h = 0.5$ and $h = 0.25$ to estimate the value of $\int_0^1 x^{(x^3)}\,dx$.

Hence give the value of the integral to the accuracy that appears justified. [5]

Further estimates of the integral using the trapezium rule for different values of h are given in the following table.

h	T
1	1
0.5	0.958 502
0.25	0.945 321
0.125	0.941 606
0.062 5	0.940 643

(iv) Obtain four Simpson's Rule estimates of the integral. Give your answers correct to six decimal places. Hence give the value of the integral to the accuracy that appears justified. [5]

(v) State the theoretical value for the ratio of differences of a sequence of estimates to a definite integral using Simpson's Rule. Use this value to obtain an improved approximation to $\int_0^1 x^{(x^3)}\,dx$. State the value of the integral as accurately as you can. [4]

7 (i) Show that the equation $3x^5 - 5x^3 - 1 = 0$ has a root α, such that $1 < \alpha < 2$, and a root β such that $-1 < \beta < 0$. [3]

(ii) Obtain the Newton-Raphson iteration

$$x_{r+1} = x_r - \frac{3x_r^5 - 5x_r^3 - 1}{15x_r^4 - 15x_r^2}. \qquad (*) \quad [2]$$

(iii) Use (*) with $x_0 = 2$ to obtain the value of α correct to 5 decimal places. [3]

(iv) Explain why it is not possible to use (*) with $x_0 = 0$ or $x_0 = -1$ to obtain a value for β. [2]

(v) Show numerically that using (*) with $x_0 = -0.3$ leads to a third root, γ. Obtain the value of γ correct to 5 significant figures. [3]

(vi) Given that $-0.9 < \beta < -0.3$, use interval bisection to obtain a value for β with a maximum possible error of 0.01875. [4]

(vii) How many further applications of bisection are needed to obtain a value for β with a maximum possible error of less than 5×10^{-5}? [1]

END OF QUESTION PAPER