

OCR

Oxford Cambridge and RSA

Tuesday 19 June 2018 – Afternoon

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

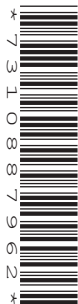
OCR supplied materials:

- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Section A (36 marks)

- 1 A point P moves round the curve with equation $3x^2 + 4y^2 = 4$. At time t , P is at the point (x, y) , as shown in Fig. 1.

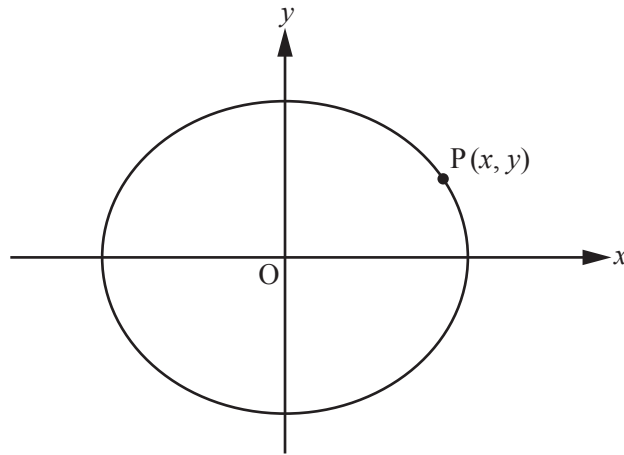


Fig. 1

- (i) Find $\frac{dy}{dx}$ in terms of x and y . [2]

- (ii) When P is at the point on the curve with x -coordinate 1 and positive y -coordinate, $\frac{dx}{dt} = 4$.

Find $\frac{dy}{dt}$ at this point. [4]

- 2 The three functions $f(x)$, $g(x)$ and $h(x)$ are defined as follows:

$$f(x) = \frac{x}{1-2x^2}, \quad g(x) = 1 + \sin 2x \quad \text{and} \quad h(x) = 3e^{-2x^2}.$$

In the table in the Answer Book, write Yes or No in each space to indicate whether the function is odd, whether it is even, and whether it is periodic. If a function is periodic, state its period. [4]

- 3 The mass of a radioactive material decreases exponentially. Its *half-life* is the time required for the mass of the material to reduce to half its initial value. The half-life of plutonium 241 is 14.4 years.

- (i) Write down the percentage of the initial mass of plutonium 241 remaining after 28.8 years. [1]

- (ii) The mass M grams of plutonium 241 at time t years is given by the equation

$$M = M_0 e^{-kt},$$

where M_0 grams is the initial mass and k is a constant. Find k , giving your answer correct to two significant figures. [3]

- 4 Fig. 4 shows part of the curve with equation $y = e^{-x} \sin 2x$.

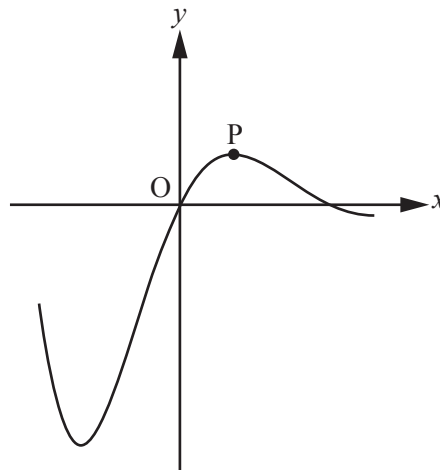


Fig. 4

Find the coordinates of the maximum point P. [6]

- 5 (i) On the same axes, sketch the graphs of $y = -|x + 1|$ and $y = 2x$. [3]

(ii) Solve the equation $-|x + 1| = 2x$. [2]

- 6 The function $h(x)$ is such that $h(x) = fg(x)$, where $f(x) = 2x + \frac{1}{2}\pi$ for $x \in \mathbb{R}$ and $g(x) = \arcsin x$ for $-1 \leq x \leq 1$.

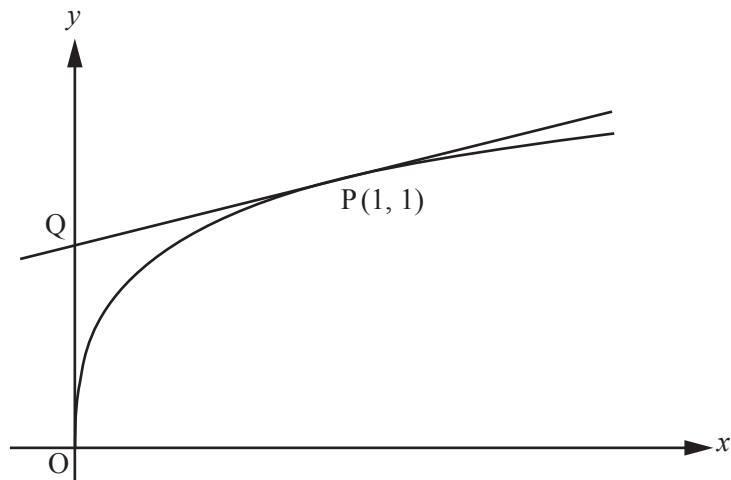
(i) Find $h\left(\frac{1}{2}\right)$, giving your answer as a multiple of π . [2]

(ii) Find $h^{-1}(x)$. [4]

- 7 Prove that $n^3 - 3n^2 + 2n$ is divisible by 6 for all positive integers n . [5]

Section B (36 marks)

- 8 Fig. 8 shows the curve with equation $y = \frac{2\sqrt{x}}{1+\sqrt{x}}$. The tangent to the curve at P (1, 1) intersects the y-axis at Q.

**Fig. 8**

- (i) Show that $\frac{dy}{dx} = \frac{1}{\sqrt{x}(1+\sqrt{x})^2}$.

Hence find the equation of PQ, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. [7]

- (ii) Show that the substitution $u = 1 + \sqrt{x}$ transforms $\int \frac{2\sqrt{x}}{1+\sqrt{x}} dx$ to $\int \frac{4(u-1)^2}{u} du$. [3]

- (iii) Hence find the exact area of the region enclosed by the curve, the y-axis and the line PQ. [8]

- 9 Fig. 9 shows the curves with equations $y = \ln x$ and $y = 2 \ln(x-2)$ which intersect at Q. The curve $y = 2 \ln(x-2)$ crosses the x -axis at P.

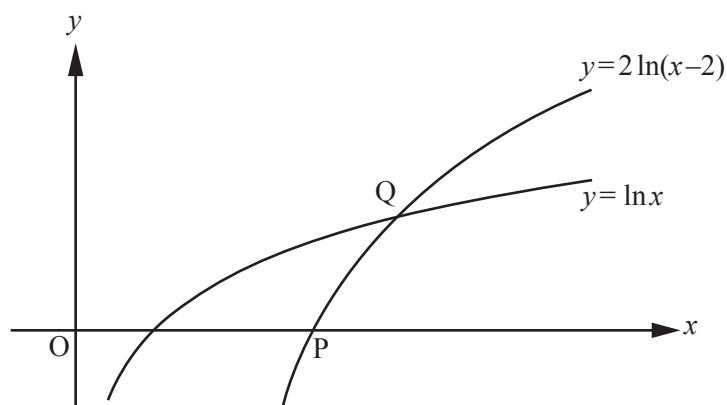


Fig. 9

- (i) Describe a sequence of two transformations which maps the curve $y = \ln x$ onto the curve $y = 2 \ln(x-2)$. [3]
- (ii) Find the exact coordinates of P and Q. [5]
- (iii) Using integration by parts, show that $\int \ln x \, dx = x \ln x - x + c$, where c is an arbitrary constant. [3]
- (iv) Hence show that the area of the finite region enclosed by the curve $y = \ln x$, the curve $y = 2 \ln(x-2)$ and the x -axis is $m \ln 2 + n$, where m and n are integers to be determined. [7]

END OF QUESTION PAPER

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