

# OCR

Oxford Cambridge and RSA

## Monday 25 June 2018 – Morning

### A2 GCE MATHEMATICS (MEI)

**4756/01** Further Methods for Advanced Mathematics (FP2)

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

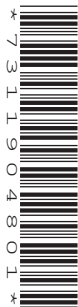
**OCR supplied materials:**

- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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**Section A (54 marks)**

- 1 (a) The polar equation of a curve is  $r = a \sin^2 \theta \cos \theta$  for  $0 \leq \theta \leq \frac{1}{2}\pi$ .

(i) Find the value of  $\theta$  for which the curve has the maximum  $x$ -coordinate. [3]

(ii) Prove that the maximum  $y$ -coordinate on the curve is  $\frac{3\sqrt{3}}{16}a$  and state the value of  $\theta$  at which this is attained. [4]

(b) (i) Sketch the graph of  $y = \arcsin x$  for  $-1 \leq x \leq 1$ . [1]

(ii) Prove that  $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$ . [4]

(iii) Using integration by parts and a suitable substitution, show that

$$\int_0^1 x^2 \arcsin x \, dx = \frac{3\pi - 4}{18}. \quad [6]$$

- 2 (a) (i) Use de Moivre's theorem to prove that

$$\cot 4\theta = \frac{1 - 6 \tan^2 \theta + \tan^4 \theta}{4 \tan \theta (1 - \tan^2 \theta)}. \quad [5]$$

(ii) Hence express the roots of the equation

$$x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$$

in exact trigonometrical form. [4]

- (b) The vertices of a square with sides of length 1 unit lie on the axes of an Argand diagram. The vertices represent the complex numbers  $z_1, z_2, z_3$  and  $z_4$  and the midpoints of the sides of the square represent the complex numbers  $z_5, z_6, z_7$  and  $z_8$ .

(i) Express  $z_5, z_6, z_7$  and  $z_8$  in modulus-argument form, and hence determine a polynomial equation of degree 4, with integer coefficients, whose roots are  $z_5, z_6, z_7$  and  $z_8$ . [4]

Let  $P(z) = 0$  be a polynomial equation of degree 8, with integer coefficients, whose roots are  $z_1, z_2, z_3, z_4, z_5, z_6, z_7$  and  $z_8$ .

(ii) Explain why  $P(z)$  cannot be of the form  $az^8 + b$  where  $a$  and  $b$  are integers. [1]

(iii) Find  $P(z)$ . [4]

- 3 (i) Find the inverse of the matrix  $\begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & k \\ k & 1 & 6 \end{pmatrix}$ . [5]

The matrix  $\mathbf{M}$  has eigenvalues 1, 2 and 3. The corresponding eigenvectors are  $\begin{pmatrix} 2 \\ 0 \\ 6 \end{pmatrix}$ ,  $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$  respectively.

- (ii) Write down the matrix  $\mathbf{P}$  such that  $\mathbf{M} = \mathbf{PDP}^{-1}$  where  $\mathbf{D} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ . [2]

- (iii) Hence find  $\mathbf{M}$ . [5]

- (iv) Find constants  $a$ ,  $b$  and  $c$  such that  $\mathbf{M}^{-1} = a\mathbf{M}^2 + b\mathbf{M} + c\mathbf{I}$ . [6]

### Section B (18 marks)

- 4 (i) Prove, using definitions in terms of exponential functions, that

$$\cosh 2A = 1 + 2 \sinh^2 A. \quad [3]$$

- (ii) Find  $\int \sinh^2 x \, dx$ . [3]

- (iii) Let  $z = \operatorname{arsinh}(1)$ . Form an equation involving  $z$  and solve it to find the exact value of  $\operatorname{arsinh}(1)$  in logarithmic form. [4]

- (iv) Using a substitution of the form  $ax = b \sinh u$ , find the exact value of

$$\int_0^{\frac{2}{3}} \frac{x^2}{\sqrt{4+9x^2}} \, dx,$$

giving your answer in the form  $p(q - \ln r)$ , where  $p$ ,  $q$  and  $r$  are constants. [8]

**END OF QUESTION PAPER**

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