OCR
Oxford Cambridge and RSA

Friday 15 June 2018 – Afternoon

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

DC (LK) 131833/3

1 In this question, you may assume that $t^k e^{-t} \to 0$ as $t \to \infty$ for any constant k.

The differential equation $4\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 9x = f(t)$ is to be solved for $t \geqslant 0$.

Firstly consider the case $f(t) = 9t^2 - 3t - 1$.

(i) Find the general solution for x in terms of t. **[9]**

You are given that $x = 5$ and $\frac{dx}{dt} = 0$ when $t = 0$.

(ii) Find the particular solution. **[4]**

(iii) Show that x is positive for all values of $t \geqslant 0$. **[3]**

Now consider the case $f(t) = -48\sin 2t - 14\cos 2t$.

(iv) Find the general solution for x in terms of t. **[6]**

(v) Describe the behaviour of x for large values of t. **[2]**

2 **Take g as 10 in this question.**

A particle P of mass 0.1 kg is in a liquid and is projected vertically downwards. At time t s, the velocity of P is v m s^{-1} and the depth of P below its point of projection, O, is x m. The only forces on P are its weight and a resistance force R N. A scientist investigates two different models for R.

In the first model, the resistance is given by $R = 0.2v$ and the initial speed of P is 2 m s^{-1}.

(i) Use this information to form a differential equation involving v and t. Solve the differential equation to show that $v = 5 - 3e^{-2t}$. **[7]**

(ii) Sketch the graph of v against t. **[2]**

(iii) Find an expression for x in terms of t and hence find the depth of P below O when its speed is three-quarters of its terminal speed. **[7]**

In the second model, the resistance is given by $R = 0.0625v^2$ and the initial speed of P is again 2 m s^{-1}.

(iv) Find v in terms of x. **[6]**

(v) State the terminal speed of P and find the depth of P below O when its speed is three-quarters of its terminal speed. **[2]**

3 **(a)** A curve in the x-y plane satisfies the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{2y}{x} = x^k \sin 2x$,

where k is a constant and $x > 0$.

Firstly consider the case $k = 3$.

(i) Find the general solution for y in terms of x. **[7]**

(ii) Given that $y = 0$ when $x = \frac{1}{4}\pi$, find the exact value of y when $x = \frac{1}{2}\pi$. **[4]**

Now consider the case $k = 2.5$.

(iii) Use Euler's method, with a step length of 0.1 and initial conditions $y = 0$ when $x = 0.5$, to estimate y when $x = 0.8$. The algorithm is given by $x_{r+1} = x_r + h$, $y_{r+1} = y_r + hy'_r$. **[5]**

(b) Solutions of the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 - y$ are to be investigated using a tangent field.

(i) Show that the isocline for which $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$ is a parabola. State the coordinates of its turning point. **[2]**

(ii) In your Answer Book, sketch on the given axes the isoclines for the cases $\frac{\mathrm{d}y}{\mathrm{d}x} = m$ for $m = 0, \pm 1, \pm 2$. Use these isoclines to draw a tangent field. **[3]**

(iii) Sketch the solution curve through (0, 1) and the solution curve through (1, 0). **[3]**

4 The simultaneous differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 7x + 2y + 13\mathrm{e}^{4t},$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -9x + y + \mathrm{e}^{7t}$$

are to be solved.

(i) Eliminate x to obtain a second order differential equation for y in terms of t. Hence find the general solution for y. **[12]**

(ii) Given that $y = -3$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 60$ when $t = 0$, find the particular solution for y. **[4]**

(iii) Find the corresponding particular solution for x. **[2]**

(iv) Find the smallest positive value of t for which $y = 0$. **[4]**

(v) Show that $\frac{y}{x} \to 0$ as $t \to \infty$. **[2]**

END OF QUESTION PAPER